3°)

Ex 3 –

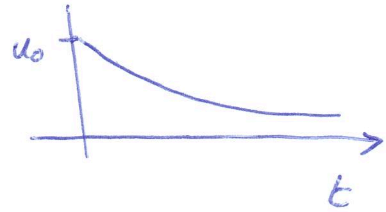
1. $\frac{d\vec{x}}{dt} = \vec{F}(\vec{x}(t), t)$ avec $\vec{x} = \begin{pmatrix} u \\ v \end{pmatrix}$, $F(\vec{x}, t) = \begin{pmatrix} v \\ -ku - av \end{pmatrix}$

2. $m\ddot{u} + a\dot{u} + ku = 0 \iff \ddot{u} + \frac{a}{m}\dot{u} + \frac{k}{m}u = 0$

3. $r^2 + \frac{a}{m}r + \frac{k}{m} = 0$, $\Delta = \left(\frac{a}{m}\right)^2 - \frac{4k}{m}$ (Rem : $\sqrt{\Delta} < \frac{a}{m}$)

- $\boxed{\Delta > 0}$ $r_{1/2} = \frac{-a/m \pm \sqrt{\Delta}}{2}$, r_1 et $r_2 < 0$

$$u(t) = A e^{r_1 t} + B e^{r_2 t}$$

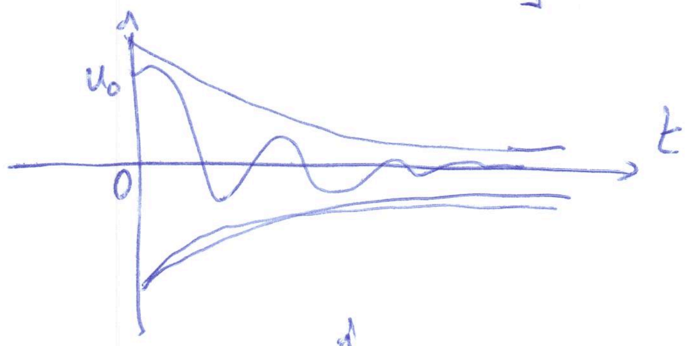


- $\boxed{\Delta < 0}$ $r_{1/2} = \frac{-a/m \pm i\sqrt{-\Delta}}{2}$

Solutions complexes : $u(t) = A e^{r_1 t} + B e^{r_2 t}$

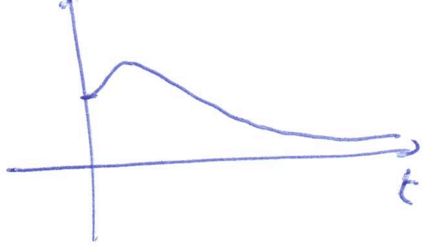
Solutions réelles : on prend $B = \overline{A}$

$$\rightarrow u(t) = \left[A' \cos\left(\frac{\sqrt{-\Delta}}{2} t\right) + B' \sin\left(\frac{\sqrt{-\Delta}}{2} t\right)\right] . e^{-\frac{at}{2m}}$$



- $\boxed{\Delta = 0}$ Racine double

$$u(t) = (At + B) e^{-\frac{at}{2m}}$$



4 – $\boxed{a = 0}$: le système est conservatif.

$E = \frac{1}{2} m\dot{u}^2 + \frac{1}{2} k u^2$ est conservé.

$u(t) = A \cos\left(\frac{\sqrt{-\Delta} t}{2}\right) + B \sin\left(\frac{\sqrt{-\Delta} t}{2}\right)$.

$\Delta < 0$ et système non asymptotiquement stable.

(oscillant).