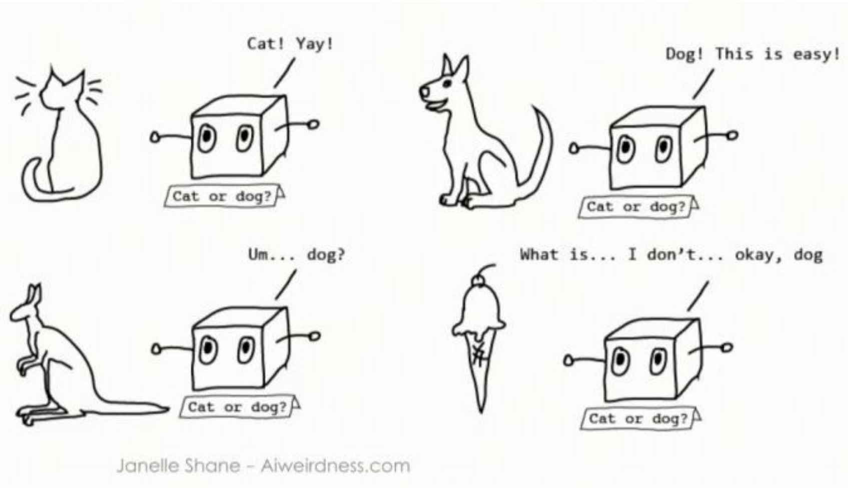
Cat! Yay!
Cat or dog?
Dog! This is easy!
Cat or dog?
Um... dog?
Cat or dog?
What is... I don't... okay, dog
Cat or dog?
Janelle Shane - Aiweirdness.com

Uncertainty reasoning and machine learning

Uncertainty, Decision and Evaluation in Machine Learning

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AOS4 master courses

Outline

- Imprecise Dirichlet Model (IDM)
  - Frequentist and Bayesian Approaches
  - Imprecise Dirichlet Model
- Applications in classification tasks
- Evaluate Classifiers

Outline

- Imprecise Dirichlet Model (IDM)
  - Frequentist and Bayesian Approaches
  - Imprecise Dirichlet Model
- Applications in classification tasks
- Evaluate Classifiers

The Importance of Sample Size (Exercise 1)

Coin	Small	Large
Flips	2	$2 \cdot 10^6$
Heads	50%	50%
Tails	50%	50%

- For both coins, a frequentist says

$$p_{\text{Heads}} = p_{\text{Tails}} = {}^1\!/_2$$

- Do Bayesians say the same thing?

The Importance of Sample Size (Exercise 1)

Coin	Small	Large
Flips	2	$2 \cdot 10^6$
Heads	50%	50%
Tails	50%	50%

- For both coins, a frequentist says

$$p_{\text{Heads}} = p_{\text{Tails}} = 1/2$$

- Do Bayesians say the same thing?

Coin	Small	Large
Flips	4	$4 \cdot 10^6$
Heads	25%	25%
Tails	75%	75%

- For both coins, a frequentist says

$$p_{\text{Heads}} = 0.25, p_{\text{Tails}} = 0.75$$

- Do Bayesians say the same thing?

The Importance of Sample Size (Exercise 1)

Coin	Small	Large
Flips	2	$2 \cdot 10^6$
Heads	50%	50%
Tails	50%	50%

- For both coins, a frequentist says

$$p_{\text{Heads}} = p_{\text{Tails}} = 1/2$$

- Do Bayesians say the same thing?

Coin	Small	Large
Flips	4	$4 \cdot 10^6$
Heads	25%	25%
Tails	75%	75%

- For both coins, a frequentist says

$$p_{\text{Heads}} = 0.25, p_{\text{Tails}} = 0.75$$

- Do Bayesians say the same thing?

Advocators	α_x	s	p^S_{H}	p^S_{T}	p^L_{H}	p^L_{T}
Haldane (1948)	0	0	???	???	???	???
Perks (1947)	$1/\lvert\mathcal{V}\rvert$	1	???	???	???	???
Jeffreys (1946, 1961)	$1/2$	$\lvert\mathcal{V}\rvert/2$	???	???	???	???
Bayes-Laplace	1	$\lvert\mathcal{V}\rvert$	???	???	???	???

The Importance of Sample Size (Solution 1)

Coin	Small	Large
Flips	2	$2 \cdot 10^6$
Heads	50%	50%
Tails	50%	50%

- For both coins, a frequentist says

$$p_{\text{Heads}} = p_{\text{Tails}} = 1/2$$

- Do Bayesians say the same thing? $\longleftarrow$
  Yes!

Coin	Small	Large
Flips	4	$4 \cdot 10^6$
Heads	25%	25%
Tails	75%	75%

- For both coins, a frequentist says

$$p_{\text{Heads}} = 0.25, p_{\text{Tails}} = 0.75$$

- Do Bayesians say the same thing?

The Importance of Sample Size (Solution 1)

Coin	Small	Large
Flips	2	$2 \cdot 10^6$
Heads	50%	50%
Tails	50%	50%

- For both coins, a frequentist says

$$p_{\text{Heads}} = p_{\text{Tails}} = 1/2$$

- Do Bayesians say the same thing? ⟵
  Yes!

Coin	Small	Large
Flips	4	$4 \cdot 10^6$
Heads	25%	25%
Tails	75%	75%

- For both coins, a frequentist says

$$p_{\text{Heads}} = 0.25, p_{\text{Tails}} = 0.75$$

- Do Bayesians say the same thing?

Advocators	α_v	s	p_H^S	p_T^S	p_H^L	p_T^L
Haldane (1948)	0	0	0.25	0.75	0.25	0.75
Perks (1947)	$1/\lvert\mathcal{V}\rvert$	1	0.3	0.7	0.25	0.75
Jeffreys (1946, 1961)	$1/2$	$\lvert\mathcal{V}\rvert/2$	0.3	0.7	0.25	0.75
Bayes-Laplace	1	$\lvert\mathcal{V}\rvert$	0.33	0.67	0.25	0.75

The Importance of Sample Size (Exercise 2)

Coin	Small	Large
Flips	2	$2 \cdot 10^6$
Heads	0%	0%
Tails	100%	100%

- For both coins, a frequentist says

$$p_{\text{Heads}} = 0, p_{\text{Tails}} = 1$$

- Do Bayesians say the same thing?

The Importance of Sample Size (Exercise 2)

Coin	Small	Large
Flips	2	$2 \cdot 10^6$
Heads	0%	0%
Tails	100%	100%

- For both coins, a frequentist says

$$p_{\text{Heads}} = 0, p_{\text{Tails}} = 1$$

- Do Bayesians say the same thing?

Advocators	α_x	s	p_H^S	p_T^S	p_H^L	p_T^L
Haldane (1948)	0	0	???	???	???	???
Perks (1947)	$1/\lvert\mathcal{V}\rvert$	1	???	???	???	???
Jeffreys (1946, 1961)	$1/2$	$\lvert\mathcal{V}\rvert/2$	???	???	???	???
Bayes-Laplace	1	$\lvert\mathcal{V}\rvert$	???	???	???	???

The Importance of Sample Size (Solution 2)

Coin	Small	Large
Flips	2	$2 \cdot 10^6$
Heads	0%	0%
Tails	100%	100%

- For both coins, a frequentist says

$$p_{\text{Heads}} = 0, p_{\text{Tails}} = 1$$

- Do Bayesians say the same thing?

Advocators	α_x	s	p^S_{H}	p^S_{T}	p^L_{H}	p^L_{T}
Haldane (1948)	0	0	0	1	0	1
Perks (1947)	$1/\lvert\mathcal{V}\rvert$	1	0.17	0.83	$3 \cdot 10^{-7}$	$1 - 3 \cdot 10^{-7}$
Jeffreys	$1/\lvert\mathcal{V}\rvert$	1	0.17	0.83	$3 \cdot 10^{-7}$	$1 - 3 \cdot 10^{-7}$
Bayes-Laplace	1	$\lvert\mathcal{V}\rvert$	0.25	0.75	$5 \cdot 10^{-7}$	$1 - 5 \cdot 10^{-7}$

Outline

The Importance of Sample Size (Exercise 3)

Coin	Small	Large
Flips	2	$2 \cdot 10^6$
Heads	50%	50%
Tails	50%	50%

- For both coins, a frequentist says

$$\theta_{\text{Heads}} = \theta_{\text{Tails}} = {}^1\!/_2$$

- Bayesians would say the same thing
- Would IDM say the same thing?

The Importance of Sample Size (Exercise 3)

Coin	Small	Large
Flips	2	$2 \cdot 10^6$
Heads	50%	50%
Tails	50%	50%

- For both coins, a frequentist says

$$\theta_{\text{Heads}} = \theta_{\text{Tails}} = {}^1\!/_2$$

- Bayesians would say the same thing
- Would IDM say the same thing?

	$\underline{P}^S_{\text{H}}$	$\overline{P}^S_{\text{H}}$	$\underline{P}^L_{\text{H}}$	$\overline{P}^L_{\text{H}}$
$s = 1$	???	???	???	???
$s = 2$	???	???	???	???

The Importance of Sample Size (Solution 3)

Coin	Small	Large
Flips	2	$2 \cdot 10^6$
Heads	50%	50%
Tails	50%	50%

- For both coins, a frequentist says

$$\theta_{\text{Heads}} = \theta_{\text{Tails}} = 1/2$$

- Bayesians would say the same thing
- Would IDM say the same thing?

	$\underline{P}^S_{\text{H}}$	$\overline{P}^S_{\text{H}}$	$\underline{P}^L_{\text{H}}$	$\overline{P}^L_{\text{H}}$
$s=1$	0.33	0.67	$0.5-3\cdot10^{-7}$	$0.5+3\cdot10^{-7}$
$s=2$	0.25	0.75	$0.5-5\cdot10^{-7}$	$0.5+5\cdot10^{-7}$

The Importance of Sample Size (Exercise 4)

Coin	Small	Large
Flips	2	$2 \cdot 10^6$
Heads	0%	0%
Tails	100%	100%

- For both coins, a frequentist says

$$\theta_{\text{Heads}} = 0, \theta_{\text{Tails}} = 1$$

- Bayesians would say different things
- What would IDM say?

The Importance of Sample Size (Exercise 4)

Coin	Small	Large
Flips	2	$2 \cdot 10^6$
Heads	0%	0%
Tails	100%	100%

- For both coins, a frequentist says

$$\theta_{\text{Heads}} = 0, \theta_{\text{Tails}} = 1$$

- Bayesians would say different things
- What would IDM say?

Advocators	α_x	s	p^S_{H}	p^S_{T}	p^L_{H}	p^L_{T}
Haldane (1948)	0	0	0	1	0	1
Perks (1947)	$1/\lvert\mathcal{V}\rvert$	1	0.17	0.83	$3 \cdot 10^{-7}$	$1 - 3 \cdot 10^{-7}$
Jeffreys	$1/\lvert\mathcal{V}\rvert$	1	0.17	0.83	$3 \cdot 10^{-7}$	$1 - 3 \cdot 10^{-7}$
Bayes-Laplace	1	$\lvert\mathcal{V}\rvert$	0.25	0.75	$5 \cdot 10^{-7}$	$1 - 5 \cdot 10^{-7}$

The Importance of Sample Size (Exercise 4)

Coin	Small	Large
Flips	2	$2 \cdot 10^6$
Heads	0%	0%
Tails	100%	100%

- For both coins, a frequentist says

$$\theta_{\text{Heads}} = 0, \theta_{\text{Tails}} = 1$$

- Bayesians would say different things
- What would IDM say?

Advocators	α_x	s	p^S_{H}	p^S_{T}	p^L_{H}	p^L_{T}
Haldane (1948)	0	0	0	1	0	1
Perks (1947)	$1/\lvert\mathcal{V}\rvert$	1	0.17	0.83	$3 \cdot 10^{-7}$	$1 - 3 \cdot 10^{-7}$
Jeffreys	$1/\lvert\mathcal{V}\rvert$	1	0.17	0.83	$3 \cdot 10^{-7}$	$1 - 3 \cdot 10^{-7}$
Bayes-Laplace	1	$\lvert\mathcal{V}\rvert$	0.25	0.75	$5 \cdot 10^{-7}$	$1 - 5 \cdot 10^{-7}$

IDM	$\underline{P}^S_{\text{H}}$	$\overline{P}^S_{\text{H}}$	$\underline{P}^L_{\text{H}}$	$\overline{P}^L_{\text{H}}$
$s = 1$	???	???	???	???
$s = 2$	???	???	???	???

The Importance of Sample Size (Solution 4)

Coin	Small	Large
Flips	2	$2 \cdot 10^6$
Heads	0%	0%
Tails	100%	100%

- For both coins, a frequentist says

$$\theta_{\text{Heads}} = 0, \theta_{\text{Tails}} = 1$$

- Bayesians would say different things
- What would IDM say?

Advocators	α_X	s	p_{H}^S	p_{T}^S	p_{H}^L	p_{T}^L
Haldane (1948)	0	0	0	1	0	1
Perks (1947)	$1/\lvert\mathcal{V}\rvert$	1	0.17	0.83	$3 \cdot 10^{-7}$	$1 - 3 \cdot 10^{-7}$
Jeffreys	$1/\lvert\mathcal{V}\rvert$	1	0.17	0.83	$3 \cdot 10^{-7}$	$1 - 3 \cdot 10^{-7}$
Bayes-Laplace	1	$\lvert\mathcal{V}\rvert$	0.25	0.75	$5 \cdot 10^{-7}$	$1 - 5 \cdot 10^{-7}$

IDM	$\underline{P}_{\text{H}}^S$	$\overline{P}_{\text{H}}^S$	$\underline{P}_{\text{H}}^L$	$\overline{P}_{\text{H}}^L$
$s = 1$	0	0.33	0	$5 \cdot 10^{-7}$
$s = 2$	0	0.50	0	10^{-6}

Determine $\mathscr{D}$ (Exercise 5)

Coin	Small	Large
Flips	2	$2 \cdot 10^6$
Heads	$[0,1]$	$[5,10]$
Tails	$[1,2]$	$[5, 2 \cdot 10^6]$

- Recap: $\mathscr{D} = \{\boldsymbol{D} | n_v \in \boldsymbol{n}_v, \sum_{v \in \mathcal{V}} n_v = n\}$
- What is $\mathscr{D}^S$ for the first coin?
- What is $\mathscr{D}^L$ for the second coin?

Determine $\mathscr{D}$ (Exercise 5)

Coin	Small	Large
Flips	2	$2 \cdot 10^6$
Heads	$[0,1]$	$[5,10]$
Tails	$[1,2]$	$[5, 2\cdot 10^6]$

- Recap: $\mathscr{D} = \{\boldsymbol{D} | n_v \in \boldsymbol{n}_v, \sum_{v \in \mathcal{V}} n_v = n\}$
- What is $\mathscr{D}^S$ for the first coin?
- What is $\mathscr{D}^L$ for the second coin?

Coin	Small	$\boldsymbol{D}_1$	$\boldsymbol{D}_2$
Flips	2	2	2
Heads	$[0,1]$	0	1
Tails	$[1,2]$	2	1

Determine $\mathscr{D}$ (Exercise 5)

Coin	Small	Large
Flips	2	$2 \cdot 10^6$
Heads	$[0,1]$	$[5,10]$
Tails	$[1,2]$	$[5, 2 \cdot 10^6]$

- Recap: $\mathscr{D} = \{\boldsymbol{D} | n_v \in \boldsymbol{n}_v, \sum_{v \in \mathcal{V}} n_v = n\}$
- What is $\mathscr{D}^S$ for the first coin?
- What is $\mathscr{D}^L$ for the second coin?

Coin	Small	$\boldsymbol{D}_1$	$\boldsymbol{D}_2$
Flips	2	2	2
Heads	$[0,1]$	0	1
Tails	$[1,2]$	2	1

Coin	Large	$\boldsymbol{D}_1$	$\boldsymbol{D}_2$	$\boldsymbol{D}_3$	$\boldsymbol{D}_4$	$\boldsymbol{D}_5$	$\boldsymbol{D}_6$
Flips	$n = 2 \cdot 10^6$	n	n	n	n	n	n
Heads	$[5,10]$	???	???	???	???	???	???
Tails	$[5,n]$	???	???	???	???	???	???

Determine $\mathscr{D}$ (Solution 5)

Coin	Small	Large
Flips	2	$2 \cdot 10^6$
Heads	$[0,1]$	$[5,10]$
Tails	$[1,2]$	$[5, 2 \cdot 10^6]$

- Recap: $\mathscr{D} = \{\boldsymbol{D} | n_v \in \boldsymbol{n}_v, \sum_{v \in \mathcal{V}} n_v = n\}$
- What is $\mathscr{D}^S$ for the first coin?
- What is $\mathscr{D}^L$ for the second coin?

Coin	Small	$\boldsymbol{D}_1$	$\boldsymbol{D}_2$
Flips	2	2	2
Heads	$[0,1]$	0	1
Tails	$[1,2]$	2	1

Coin	Large	$\boldsymbol{D}_1$	$\boldsymbol{D}_2$	$\boldsymbol{D}_3$	$\boldsymbol{D}_4$	$\boldsymbol{D}_5$	$\boldsymbol{D}_6$
Flips	$n = 2 \cdot 10^6$	n	n	n	n	n	n
Heads	$[5,10]$	5	6	7	8	9	10
Tails	$[5,n]$	$n-5$	$n-6$	$n-7$	$n-8$	$n-9$	$n-10$

Compute Lower and Upper Expectations (Exercise 6)

Interval posterior mean $\theta_v^*|\mathscr{D}$ of $\theta_v|\mathscr{D}$:

$$\underline{E}(\theta_v|\mathscr{D}) = \min_{\boldsymbol{D}\in\mathscr{D}} \underline{E}(\theta_v|\boldsymbol{D}) = \min_{\boldsymbol{D}\in\mathscr{D}} {}^{n_v}\!/_{(n+s)}, \tag{1}$$

$$\overline{E}(\theta_v|\mathscr{D}) = \max_{\boldsymbol{D}\in\mathscr{D}} \overline{E}(\theta_v|\boldsymbol{D}) = \max_{\boldsymbol{D}\in\mathscr{D}} {}^{(n_v+s)}\!/_{(n+s)}. \tag{2}$$

Compute Lower and Upper Expectations (Exercise 6)

Interval posterior mean $\theta_v^*|\mathscr{D}$ of $\theta_v|\mathscr{D}$:

$$\underline{E}(\theta_v|\mathscr{D}) = \min_{\boldsymbol{D}\in\mathscr{D}} \underline{E}(\theta_v|\boldsymbol{D}) = \min_{\boldsymbol{D}\in\mathscr{D}} n_v/(n+s), \quad (1)$$

$$\overline{E}(\theta_v|\mathscr{D}) = \max_{\boldsymbol{D}\in\mathscr{D}} \overline{E}(\theta_v|\boldsymbol{D}) = \max_{\boldsymbol{D}\in\mathscr{D}} (n_v+s)/(n+s). \quad (2)$$

Coin	Small	$\boldsymbol{D}_1$	$\boldsymbol{D}_2$	$\underline{E}(\theta_v\|\mathscr{D})$	$\overline{E}(\theta_v\|\boldsymbol{D})$
Flips	2	2	2		
Heads	$[0,1]$	0	1	???	???
Tails	$[1,2]$	2	1	???	???

Compute Lower and Upper Expectations (Exercise 6)

Interval posterior mean $\theta_v^*|\mathscr{D}$ of $\theta_v|\mathscr{D}$:

$$\underline{E}(\theta_v|\mathscr{D}) = \min_{\boldsymbol{D}\in\mathscr{D}} \underline{E}(\theta_v|\boldsymbol{D}) = \min_{\boldsymbol{D}\in\mathscr{D}} n_v/(n+s), \quad (1)$$

$$\overline{E}(\theta_v|\mathscr{D}) = \max_{\boldsymbol{D}\in\mathscr{D}} \overline{E}(\theta_v|\boldsymbol{D}) = \max_{\boldsymbol{D}\in\mathscr{D}} (n_v+s)/(n+s). \quad (2)$$

Coin	Small	$\boldsymbol{D}_1$	$\boldsymbol{D}_2$	$\underline{E}(\theta_v\|\mathscr{D})$	$\overline{E}(\theta_v\|\boldsymbol{D})$
Flips	2	2	2		
Heads	$[0,1]$	0	1	???	???
Tails	$[1,2]$	2	1	???	???

Coin	Large	$\boldsymbol{D}_1$	...	$\boldsymbol{D}_6$	$\underline{E}(\theta_v\|\mathscr{D})$	$\overline{E}(\theta_v\|\boldsymbol{D})$
Flips	$n = 2\cdot 10^6$	n	...	n		
Heads	$[5,10]$	5	...	10	???	???
Tails	$[5,n]$	$n-5$	...	$n-10$	???	???

Compute Lower and Upper Expectations (Solution 6)

Interval posterior mean $\theta_v^*|\mathscr{D}$ of $\theta_v|\mathscr{D}$:

$$\underline{E}(\theta_v|\mathscr{D}) = \min_{\boldsymbol{D}\in\mathscr{D}} \underline{E}(\theta_v|\boldsymbol{D}) = \min_{\boldsymbol{D}\in\mathscr{D}} {}^{n_v}/_{(n+s)}, \quad (3)$$

$$\overline{E}(\theta_v|\mathscr{D}) = \max_{\boldsymbol{D}\in\mathscr{D}} \overline{E}(\theta_v|\boldsymbol{D}) = \max_{\boldsymbol{D}\in\mathscr{D}} {}^{(n_v+s)}/_{(n+s)}. \quad (4)$$

Coin	Small	$\boldsymbol{D}_1$	$\boldsymbol{D}_2$	$\underline{E}(\theta_v\|\mathscr{D})$	$\overline{E}(\theta_v\|\boldsymbol{D})$
Flips	2	2	2		
Heads	$[0,1]$	0	1	${}^0/_{(2+s)}$	${}^{(1+s)}/_{(2+s)}$
Tails	$[1,2]$	2	1	${}^1/_{(2+s)}$	${}^{(2+s)}/_{(2+s)}$

Compute Lower and Upper Expectations (Solution 6)

Interval posterior mean $\theta_v^*|\mathscr{D}$ of $\theta_v|\mathscr{D}$:

$$\underline{E}(\theta_v|\mathscr{D}) = \min_{\boldsymbol{D}\in\mathscr{D}} \underline{E}(\theta_v|\boldsymbol{D}) = \min_{\boldsymbol{D}\in\mathscr{D}} n_v/(n+s), \quad (3)$$

$$\overline{E}(\theta_v|\mathscr{D}) = \max_{\boldsymbol{D}\in\mathscr{D}} \overline{E}(\theta_v|\boldsymbol{D}) = \max_{\boldsymbol{D}\in\mathscr{D}} (n_v+s)/(n+s). \quad (4)$$

Coin	Small	$\boldsymbol{D}_1$	$\boldsymbol{D}_2$	$\underline{E}(\theta_v\|\mathscr{D})$	$\overline{E}(\theta_v\|\boldsymbol{D})$
Flips	2	2	2		
Heads	$[0,1]$	0	1	$0/(2+s)$	$(1+s)/(2+s)$
Tails	$[1,2]$	2	1	$1/(2+s)$	$(2+s)/(2+s)$

Coin	Large	$\boldsymbol{D}_1$	…	$\boldsymbol{D}_6$	$\underline{E}(\theta_v\|\mathscr{D})$	$\overline{E}(\theta_v\|\boldsymbol{D})$
Flips	$n=2\cdot 10^6$	n	…	n		
Heads	$[5,10]$	5	…	10	$5/(n+s)$	$(10+s)/(n+s)$
Tails	$[5,n]$	$n-5$	…	$n-10$	$(n-10)/(n+s)$	$(n-5+s)/(n+s)$

Outline

- Imprecise Dirichlet Model (IDM)
- Applications in classification tasks
  - Pazen Window Classifiers
- Evaluate Classifiers

Outline

- Imprecise Dirichlet Model (IDM)
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  - Pazen Window Classifiers
- Evaluate Classifiers

Determine Possible Precise Data Set (Exercise 7)

$\boldsymbol{x}' \in \boldsymbol{D}_\epsilon(\boldsymbol{x})$	$Y \subset \mathscr{Y} = \{\text{Apple}, \text{Banana}, \text{Tomato}\}$
$\boldsymbol{x}'_1$	Apple or Banana, but not Tomato
$\boldsymbol{x}'_2$	Banana or Tomato, but not Apple
$\boldsymbol{x}'_3$	Apple or Tomato, but not Banana
$\boldsymbol{x}'_4$	Tomato
$\boldsymbol{x}'_5$	Tomato
$\boldsymbol{x}'_6$	Banana
$\boldsymbol{x}'_7$	Banana

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$$n = 7, \boldsymbol{n}_A = ???, \boldsymbol{n}_B = ???, \boldsymbol{n}_T = ??? \quad (5)$$

Determine Possible Precise Data Set (Exercise 7)

$\mathbf{x}' \in \mathbf{D}_\epsilon(\mathbf{x})$	$Y \subset \mathscr{Y} = \{\text{Apple}, \text{Banana}, \text{Tomato}\}$
$\mathbf{x}'_1$	Apple or Banana, but not Tomato
$\mathbf{x}'_2$	Banana or Tomato, but not Apple
$\mathbf{x}'_3$	Apple or Tomato, but not Banana
$\mathbf{x}'_4$	Tomato
$\mathbf{x}'_5$	Tomato
$\mathbf{x}'_6$	Banana
$\mathbf{x}'_7$	Banana

7

$$n = 7, \mathbf{n}_A = ???, \mathbf{n}_B = ???, \mathbf{n}_T = ??? \tag{5}$$

	$\mathbf{D}_1$	$\mathbf{D}_2$	$\mathbf{D}_3$	$\mathbf{D}_4$	$\mathbf{D}_5$	$\mathbf{D}_6$	$\mathbf{D}_7$	$\mathbf{D}_8$
n_A	???	???	???	???	???	???	???	???
n_B	???	???	???	???	???	???	???	???
n_T	???	???	???	???	???	???	???	???

Determine Possible Precise Data Set (Solution 7)

$\boldsymbol{x}' \in \boldsymbol{D}_\epsilon(\boldsymbol{x})$	$Y \subset \mathscr{Y} = \{\text{Apple}, \text{Banana}, \text{Tomato}\}$
$\boldsymbol{x}'_1$	Apple or Banana, but not Tomato
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$\boldsymbol{x}'_3$	Apple or Tomato, but not Banana
$\boldsymbol{x}'_4$	Tomato
$\boldsymbol{x}'_5$	Tomato
$\boldsymbol{x}'_6$	Banana
$\boldsymbol{x}'_7$	Banana

$$n = 7\,, \boldsymbol{n}_A = \{0, 1, 2\}\,, \boldsymbol{n}_B = \{2, 3, 4\}\,, \boldsymbol{n}_T = \{2, 3, 4\} \tag{6}$$

Determine Possible Precise Data Set (Solution 7)

$\mathbf{x}' \in \mathbf{D}_\epsilon(\mathbf{x})$	$Y \subset \mathscr{Y} = \{\text{Apple}, \text{Banana}, \text{Tomato}\}$
$\mathbf{x}'_1$	Apple or Banana, but not Tomato
$\mathbf{x}'_2$	Banana or Tomato, but not Apple
$\mathbf{x}'_3$	Apple or Tomato, but not Banana
$\mathbf{x}'_4$	Tomato
$\mathbf{x}'_5$	Tomato
$\mathbf{x}'_6$	Banana
$\mathbf{x}'_7$	Banana

$$n = 7, \mathbf{n}_A = \{0, 1, 2\}, \mathbf{n}_B = \{2, 3, 4\}, \mathbf{n}_T = \{2, 3, 4\} \quad (6)$$

	$\mathbf{D}_1$	$\mathbf{D}_2$	$\mathbf{D}_3$	$\mathbf{D}_4$	$\mathbf{D}_5$	$\mathbf{D}_6$	$\mathbf{D}_7$	$\mathbf{D}_8$
n_A	0	0	1	1	1	2	2	2
n_B	3	4	2	3	4	2	3	4
n_T	4	3	4	3	2	4	3	3

Compute Lower and Upper Expectations (Exercise 8)

	D_1	D_2	D_3	D_4	D_5	D_6	D_7	D_8
n_A	0	0	1	1	1	2	2	2
n_B	3	4	2	3	4	2	3	4
n_T	4	3	4	3	2	4	3	3

Using IDM to estimate **interval posterior mean** $\theta_y^*|\mathscr{D}$ of $\theta_y|\mathscr{D}$:

$$\underline{E}(\theta_y|\boldsymbol{x}) = \min_{\boldsymbol{D}\in\mathscr{D}} \underline{E}(\theta_y|\boldsymbol{x}) = \min_{\boldsymbol{D}\in\mathscr{D}} {n_y}/{(n+s)}, \tag{7}$$

$$\overline{E}(\theta_y|\boldsymbol{x}) = \max_{\boldsymbol{D}\in\mathscr{D}} \overline{E}(\theta_y|\boldsymbol{D}) = \max_{\boldsymbol{D}\in\mathscr{D}} {(n_y+s)}/{(n+s)}. \tag{8}$$

Compute Lower and Upper Expectations (Exercise 8)

	D_1	D_2	D_3	D_4	D_5	D_6	D_7	D_8
n_A	0	0	1	1	1	2	2	2
n_B	3	4	2	3	4	2	3	4
n_T	4	3	4	3	2	4	3	3

Using IDM to estimate **interval posterior mean** $\theta_y^*|\mathscr{D}$ of $\theta_y|\mathscr{D}$:

$$\underline{E}(\theta_y|\boldsymbol{x}) = \min_{\boldsymbol{D}\in\mathscr{D}} \underline{E}(\theta_y|\boldsymbol{x}) = \min_{\boldsymbol{D}\in\mathscr{D}} n_y/(n+s), \tag{7}$$

$$\overline{E}(\theta_y|\boldsymbol{x}) = \max_{\boldsymbol{D}\in\mathscr{D}} \overline{E}(\theta_y|\boldsymbol{D}) = \max_{\boldsymbol{D}\in\mathscr{D}} (n_y+s)/(n+s). \tag{8}$$

	$\underline{E}(\theta_y\|\boldsymbol{x})$	$\overline{E}(\theta_y\|\boldsymbol{x})$
A	???	???
B	???	???
T	???	???

Compute Lower and Upper Expectations (Solution 8)

	D_1	D_2	D_3	D_4	D_5	D_6	D_7	D_8
n_A	0	0	1	1	1	2	2	2
n_B	3	4	2	3	4	2	3	4
n_T	4	3	4	3	2	4	3	3

Using IDM to estimate **interval posterior mean** $\theta_y^*|\mathscr{D}$ of $\theta_y|\mathscr{D}$:

$$\underline{E}(\theta_y|\boldsymbol{x}) = \min_{\boldsymbol{D}\in\mathscr{D}} \underline{E}(\theta_y|\boldsymbol{x}) = \min_{\boldsymbol{D}\in\mathscr{D}} {n_y}/{(n+s)}, \tag{9}$$

$$\overline{E}(\theta_y|\boldsymbol{x}) = \max_{\boldsymbol{D}\in\mathscr{D}} \overline{E}(\theta_y|\boldsymbol{D}) = \max_{\boldsymbol{D}\in\mathscr{D}} {(n_y+s)}/{(n+s)}. \tag{10}$$

	$\underline{E}(\theta_y|\boldsymbol{x})$	$\overline{E}(\theta_y|\boldsymbol{x})$
A	$0/(7+s)$	$(2+s)/(7+s)$
B	$2/(7+s)$	$(4+s)/(7+s)$
T	$2/(7+s)$	$(4+s)/(7+s)$

Outline

Set-Based Utility Functions (Exercise 9)

Recap: Few commonly used **utility functions**:

$$g_\alpha(|Y|) = \frac{\alpha}{|Y|} - \frac{\alpha - 1}{|Y|^2}.$$

Exercise: The maximum value of α such that $g_\alpha(|Y|) \leq 1,\ \forall Y \subset \mathscr{Y} \setminus \emptyset$?

References I