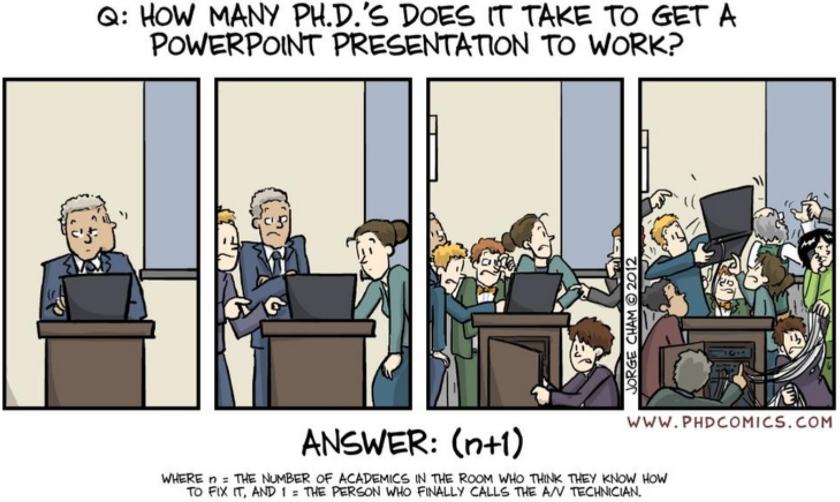
Q: HOW MANY PH.D.'S DOES IT TAKE TO GET A POWERPOINT PRESENTATION TO WORK?
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WWW.PHDCOMICS.COM
ANSWER: (n+1)
WHERE n = THE NUMBER OF ACADEMICS IN THE ROOM WHO THINK THEY KNOW HOW TO FIX IT, AND 1 = THE PERSON WHO FINALLY CALLS THE A/V TECHNICIAN.

Uncertainty reasoning and machine learning

A Few Applications

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AOS4 master courses

Outline

- Excercises on prediction-making
  - Probabilistic classifiers
  - Credal classifiers

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Bayesian-optimal predictions

A **Bayesian-optimal prediction (BOP) of** u is

$$y_d^u \in \operatorname*{argmax}_{y' \in \mathcal{Y}} \mathbf{E}\left[u(y', y)\right] = \operatorname*{argmax}_{y' \in \mathcal{Y}} \sum_{y \in \mathcal{Y}} u(y', y) \mathbf{p}_d(y). \quad (1)$$

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Question: Prove that a BOP (1) takes from $O(K)$ to $O(K^2)$.

Bayesian-optimal set-valued predictions

A **Bayesian-optimal prediction (BOP) of** U is

$$Y_d^U \in \underset{Y' \subset \mathcal{Y}}{\operatorname{argmax}} \mathbf{E}[U(Y', y)] = \underset{Y' \subset \mathcal{Y}}{\operatorname{argmax}} \sum_{y \in \mathcal{Y}} U(Y', y) \mathbf{p}_d(y). \tag{2}$$

Bayesian-optimal set-valued predictions

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Question: Prove that for any utility-discounted accuracy

$$U(Y', y) = \frac{1}{g(|Y'|)} \llbracket y \in Y' \rrbracket , \tag{3}$$

finding a BOP Y_d^U (2) takes $O(K \log(K))$.

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finding a BOP Y_d^U (2) takes $O(K \log(K))$.

Hint: First show that a BOP Y_d^U (2) consists of the most probable classes on $\mathbf{p}_d$.

Outline

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E-admissible and maximal sets (Recap)

- **E-admissibility** under u: A class y is E-admissible if there exist $\mathbf{p} \in \mathbf{CH}^{d}_{\alpha^*}$ so that $y = y^u$.
- **Maximality** under u: A class y is maximal if there doesn't exist $y' \neq y$ such that y' dominates y on all $\mathbf{p} \in \mathbf{CH}^{d}_{\alpha^*}$ (w.r.t. u).

Check if a class is *E*-admissible

Question: Prove that checking whether a given class y is E-admissible can be done by solving a linear program.

Check if a class is maximal

Question: Prove that checking whether a given class y is maximal can be done by solving $K-1$ linear program.

Properties of E-admissible and maximal sets

- **Question 1**: Prove that the E-admissible set is a subset of the maximal set.
- **Question 2**: Show that the E-admissible set can be a strict subset of the maximal set.
- **Question 3**: Show that the two sets can be identical.
- **Question 4**: Show that the cardinality of the E-admissible set can be larger than the number of extreme points on the credal set.

Properties of E-admissible and maximal sets: Hints

- **Question 1**: Prove that the E-admissible set is a subset of the maximal set. → We did it during the last lecture.
- **Question 2**: Show that the E-admissible set can be a strict subset of the maximal set. → Consider the credal set with two extreme points $\{(0.35, 0.4, 0.25), (0.3, 0.2, 0.5)\}$.
- **Question 3**: Show that the two sets can be identical. → Consider the credal set with two extreme points $\{(0.3, 0.5, 0.2), (0.2, 0.7, 0.1)\}$
- **Question 4**: Show that the cardinality of the E-admissible set can be larger than the number of extreme points on the credal set. → Consider the credal set with two extreme points $\{(0.6, 0.4, 0.0), (0.0, 0.4, 0.6)\}$

References I