

# Multiobjective Genetic Algorithms for Design of Water Distribution Networks

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**Abstract:** This paper presents a multiobjective genetic algorithm approach to the design of a water distribution network. The objectives considered are minimization of the network cost and maximization of a reliability measure. In this study, a new reliability measure, called network resilience, is introduced. This measure mimics a designer's desire of providing excess head above the minimum allowable head at the nodes and of designing reliable loops with practicable pipe diameters. The proposed method produces a set of Pareto-optimal solutions in the search space of cost and network resilience. Genetic algorithms are observed to be poor in handling constraints. To handle constraints in a better way, a constraint handling technique that does not require a penalty coefficient and is applicable to water distribution systems is presented. The present model is applied to two example problems, which are widely reported. Comparison of the present method with other methods revealed that the network resilience based approach gave better results.

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## Introduction

When a source of water is far off demand points, water has to be transmitted through a network of pipes from the source to demand points. The present day water distribution networks are complex and require huge investments in their construction and maintenance. For these reasons, a need to improve their efficiency by way of minimizing their cost and maximizing the benefit accrued from them is strongly felt. In the past, design of a water distribution network was based on experience. However, in the last three decades, a significant number of methods have been developed using linear programming, dynamic programming, enumeration techniques, heuristic methods, and evolutionary techniques (Alperovits and Shamir 1977; Quindry et al. 1981; Gessler and Walski 1985; Goulter and Morgan 1985; Duan et al. 1990; Fujiwara and Khang 1990; Simpson et al. 1994; Savic and Walters 1997; Vairavamoorthy and Ali 2000). Most of these methods consider the minimization of cost of a pipe network as the objective, although some reliability studies and stochastic modeling of demands have been attempted (Goulter and Bouchart 1990; Xu and Goulter 1999). Of all the preceding methods, genetic algorithm (GA) based methods appear to be robust, as they can handle discrete pipe sizes with ease and produce a set of promising solutions.

Most of the pipe network optimization methods have not con-

sidered the layout optimization along with the cost due to the extreme complexity involved and because layout is largely restricted by the location of roads. Therefore, for a given network layout and demands, the pipe network optimization problem has been considered as the selection of pipe sizes that will minimize the cost of a network. In addition to cost, obviously, there are other possible objectives such as reliability, redundancy, and water quality that can be included in the optimization process. Quindry et al. (1981) and Goulter and Morgan (1985) have shown that networks designed by cost minimization and for a single loading condition resulted in branched networks. Stanic et al. (1998) and Abebe and Solomatine (1998) have also demonstrated this tendency of single objective optimization algorithms. Branched water distribution networks will have severe consequences in terms of reliability under failure conditions. In order to reduce their risk of failure to supply, often designers introduce redundancy into networks by adding pipes to close loops. For this purpose, many researchers have used a minimum diameter constraint, causing some of the pipes to be of an allowed minimum diameter. It must be emphasized here that the loops in a network are provided to increase its reliability, so that the system will have sufficient capacity to deliver during mechanical and/or hydraulic failures. Mechanical failures are the failure of network components such as pumps, pipe breakage, etc., and hydraulic failures are changes in demand or pressure, aging of pipe, etc. (Mays 1996). To improve the performance of a water distribution network under failure conditions, Goulter and Bouchart (1990) have solved a reliability constrained least cost optimization problem. However, explicit consideration of reliability in an optimization model is a difficult and complex task, and there are no universally accepted definitions for reliability (Mays 1996; Todini 2000).

Walski (2001), in a recent editorial, stressed the need for the development of new models, which addresses not only minimization of network cost but also maximization of net benefits. The biggest hurdle faced in water distribution network design is predicting future demands. Thus, a designer would like to provide as much excess head above the minimum allowable head at the

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nodes of a water distribution system as possible, subject to monetary constraints. The surplus head is then utilized to overcome increased head losses under increased demand or failure conditions. Walski (2001) also stressed the need for developing methods that produce reliable or practicable loops and avoid loops having pipes of widely different diameters such as 16 in. pipe connected to 2 in. pipe. These aspects strongly motivate a researcher to include other legitimate objectives such as measures of reliability into the objective function in addition to cost.

Gessler and Walski (1985) were probably the first to use a benefit function in pipe network optimization in their *WADISO* computer program. In their model, the benefit was measured as the amount of excess pressure above the minimum required at the worst node in the system over all of the loading tested (Walski and Gessler 1999). Halhal et al. (1997) were the first to use a multiobjective genetic algorithm to solve water distribution network rehabilitation problems. They considered minimization of network cost and maximization of benefit as the objectives. Benefit in this model was calculated as the summation of hydraulic benefit, physical integrity benefit, flexibility benefit, and quality benefit with each component given a weight. Here, the hydraulic benefit is quantified as the difference between the pressure deficiencies in the initial network before improvement and in the solution obtained. They used the structured messy genetic algorithm (SMGA) to solve the optimization problem. Walters et al. (1999) applied the preceding algorithm to solve the “Anytown” distribution network. Todini (2000) presented a heuristic method considering cost function and resilience index, a reliability measure, as the objectives. This method solves for minimum cost networks, heuristically, by fixing a value of resilience index between 0 and 1. More recently, Wu et al. (2002) presented a multiobjective model with cost and benefit functions as objectives. In this model, the benefit was calculated as the summation of total flow delivered at each demand node. Here total flow is modeled as the sum of a baseline demand and an emitter flow. Although these methods are a step forward in considering multiple objectives in pipe network optimization, they do not incorporate the effect of redundancy in the benefit function. Halhal et al. (1997) have included flexibility benefit—a measure of redundancy—in their benefit function, but it requires a weight to add to the total benefit. Thus, there is a need for the method to be further developed using better reliability measures and multiobjective algorithms.

When more than one objective is present in an objective function, there may not exist one solution that is best with respect to all objectives. Instead in a multiobjective optimization problem there exists a set of solutions called Pareto-optimal solutions or nondominated solutions (Hans 1988). These solutions are superior to the rest of solutions in the search space when all objectives are considered, but are inferior to other solutions in the space in one or more objectives. It must be recognized that optimization can only assist the engineer and that engineering judgment and experience is still required to provide a practicable solution. The Pareto set gives an engineer more flexibility in the selection of a practicable solution. The classical way of solving multiobjective problems is to scale the vector of objectives into one objective and solve for the optimal solution. This process results in a solution that is largely dependent on the weight vector used in the scaling process. Most of these drawbacks can be eliminated using multiobjective genetic algorithms. Therefore, in this study a multiobjective genetic algorithm, called the nondominated sorting genetic algorithm (NSGA) (Srinivas and Deb 1994), is used to obtain a Pareto-front. In the present model, the objective function consists of minimization of network cost and maximization of

network resilience, which is a measure of reliability. NSGA was observed to converge towards the true Pareto-front and distribute solutions along the front uniformly (Deb 1999). This gives a designer a number of alternatives that are superior to the rest of solutions in the search space in a multiobjective sense. The model was applied to two example problems, which were previously reported. From the analysis of the results, it is observed that use of network resilience in the objective function gave better results in terms of both surplus power and redundancy. Comparison of the present method with the other reliability measures shows the superiority of the proposed reliability measure. Here it must be mentioned that maximization of network resilience improves the reliability of network solutions but does not guarantee the delivery of water at different nodes under a failure condition. Engineering judgment coupled with reliability assessment methods such as that by Xu and Goulter (1998) and Tolson et al. (2001) can be used to select a solution from the Pareto set.

## Formulation of the Model

The following is the proposed two-objective optimization model for a water distribution network design. The two objective functions are (1) minimization of network cost; and (2) maximization of a reliability measure

$$\text{Minimize } f_1 = \sum_{i=1}^{np} C_i(D_i, L_i) \quad (1)$$

$$\text{Maximize } f_2 = I_n \quad (2)$$

where  $C_i(D_i, L_i)$  = cost of the pipe  $i$  with diameter  $D_i$  and length  $L_i$ ;  $np$  = number of pipes in the system; and  $I_n$  = network resilience. The preceding optimization model is subjected to the following constraints:

$$g_j(H, D) = 0 \quad j = 1, 2, \dots, nn \quad (3)$$

$$H_j \geq H_j^l \quad j = 1, 2, \dots, nn \quad (4)$$

$$D_i \in \{A\} \quad i = 1, 2, \dots, np \quad (5)$$

where  $nn$  = number of junction nodes;  $g(H, D)$  = nodal mass balance and loop (path) energy balance equations;  $H_j$  = head at any node  $j$ , which must be greater than a minimum value  $H_j^l$ ; and all  $D_i$ 's = discrete pipe sizes selected from a set of commercially available sizes.

The above-formulated model is a multiobjective mixed integer nonlinear optimization model. It can be solved using a multiobjective genetic algorithm. A major problem associated with GA optimization is the poor ability of GAs to handle constraints. Therefore, in general, when GAs are applied to a water distribution network design, some of the constraints such as nodal mass balance equations and energy conservation equations are satisfied externally by using a hydraulic network solver. In this study, network hydraulic analysis is performed using EPANET hydraulic solver.

## Reliability Measures

Least cost design of looped networks under a single loading condition resulted in some of the pipes having a minimum diameter and heads at some of the nodes being barely satisfied. This situation is improved by designing a network to satisfy the constraints under many critical loadings. Identification of various critical loadings is a complex process for large networks, and

evolutionary algorithms require numerous hydraulic simulations during the optimization process. These aspects encourage the use of a second approach—multiobjective optimization—that seeks for a set of solutions with improved reliability (Halhal et al. 1997; Walters et al. 1999; Todini 2000). In this approach, the objectives are (1) to minimize network cost; and (2) to maximize a reliability measure. Solution of this model with either a single loading or a few critical loadings gives a set of Pareto-optimal solutions.

Whenever there is a mechanical or hydraulic failure, the internal head losses will increase causing failure of the network. These increased head losses during failure conditions can be met, if sufficient excess power is available for internal dissipation. Based on this premise, the following reliability measures are defined.

### Minimum Surplus Head Index ( $I_m$ )

The surplus head at a node is equal to the difference between the actual head  $H$  at which the demand  $Q$  is supplied and the minimum required head or design head  $H^l$  at that node. This surplus head indicates the available energy for dissipation during failure conditions. Maximization of the available surplus head at the most depressed node improves the reliability of a network to some extent. Accordingly the minimum surplus head index  $I_m$  is defined as

$$I_m = \min\{H_j - H_j^l\} \quad j = 1, 2, \dots, nn \quad (6)$$

This index was used as the indicator of benefits in the WADISO computer program (Walski and Gessler 1999).

### Total Surplus Head Index ( $I_t$ )

Another index that can be used to measure the reliability of a network is the summation of surplus head at each junction node. In mathematical form, the total surplus head index,  $I_t$ , can be expressed as

$$I_t = \sum_{j=1}^{nn} (H_j - H_j^l) \quad \text{for all } j = 1, 2, \dots, nn \quad (7)$$

Maximization of  $I_t$  also improves the ability of a network to adjust under stressed conditions.

### Resilience Index ( $I_r$ )

Todini (2000) proposed the following resilience index, based on the concept that the power input into a network is equal to the power lost internally to overcome the friction plus the power that is delivered at demand points:

$$P_{\text{inp}} = P_{\text{int}} + P_{\text{out}} \quad (8)$$

The total input power into a network including power supplied by pumps is given by

$$P_{\text{inp}} = \gamma \sum_{k=1}^{nr} Q_k H_k + \sum_{i=1}^{npu} P_i \quad (9)$$

where  $Q_k$  and  $H_k$  = discharge and head corresponding to each reservoir node  $k$ ;  $nr$  = number of reservoir nodes;  $P_i$  = power supplied by pump  $i$ ; and  $npu$  = number of pumps in a network. The total output power is given by

$$P_{\text{out}} = \gamma \sum_{j=1}^{nn} Q_j H_j \quad (10)$$

where  $Q_j$  = demand at node  $j$ ; and  $H_j$  = head at which  $Q_j$  is supplied. The resilience index of a network is then defined as

$$I_r = 1 - \left( \frac{P_{\text{int}}}{P_{\text{int}}^{\text{max}}} \right) \quad (11)$$

where  $P_{\text{int}}$  = amount of power dissipated in a network; and  $P_{\text{int}}^{\text{max}}$  = maximum power that would be dissipated internally in order to satisfy design demand  $Q$  and design head  $H^l$  at the junction nodes. Substitution of appropriate quantities in Eq. (11) gives

$$I_r = \frac{\sum_{j=1}^{nn} Q_j (H_j - H_j^l)}{(\sum_{k=1}^{nr} Q_k H_k + \sum_{i=1}^{npu} P_i / \gamma) - \sum_{j=1}^{nn} Q_j H_j^l} \quad (12)$$

Maximization of the resilience index also improves the ability of a pipe network to counter the failure conditions.

### Network Resilience ( $I_n$ )

Maximization of the preceding three indices may increase surplus head or power at junction nodes, but they do not reflect the effect of redundancy. A branched network with sufficient surplus head at the nodes may adjust to increased demands, but a pipe outage will have severe consequences at one or more downstream nodes. Therefore, maximization of surplus head or power alone is not sufficient for a reliable network. The following reliability measure, called network resilience ( $I_n$ ), incorporates the effects of both surplus power and reliable loops. The surplus power at any node  $j$  is given by

$$P_j = \gamma Q_j (H_j - H_j^l) \quad (13)$$

Reliable loops can be ensured, if the pipes connected to a node are not widely varying in diameter. If  $D_1$ ,  $D_2$ , and  $D_3$  (where  $D_1 \geq D_2 \geq D_3$ ) are the diameters of three pipes connected to node  $j$ , then uniformity of that node is given by

$$C_j = \frac{(D_1 + D_2 + D_3)}{3D_1} \quad (14)$$

and in generalized form

$$C_j = \frac{\sum_{i=1}^{npj} D_i}{npj \times \max\{D_i\}} \quad (15)$$

where  $npj$  = number of pipes connected to node  $j$ . The value of  $C = 1$ , if pipes connected to a node have the same diameter; and  $C < 1$ , if pipes connected to a node have different diameters. For nodes connected with only one pipe, the value of  $C$  is taken to be one. The combined effect of both surplus power and nodal uniformity of node  $j$ , called weighted surplus power, is expressed as

$$X_j = C_j P_j$$

For a network, it is given by

$$X = \sum_{j=1}^{nn} X_j = \sum_{j=1}^{nn} C_j P_j = \sum_{j=1}^{nn} C_j Q_j (H_j - H_j^l) \quad (16)$$

Eq. (16) may be normalized by dividing with maximum surplus power to get network resilience as

$$I_n = \frac{X}{X_{\text{max}}} = \frac{\sum_{j=1}^{nn} C_j Q_j (H_j - H_j^l)}{[\sum_{k=1}^{nr} Q_k H_k + \sum_{i=1}^{npu} (P_i / \gamma)] - \sum_{j=1}^{nn} Q_j H_j^l} \quad (17)$$

where  $X_{\text{max}} (= P_{\text{inp}} - \sum_{j=1}^{nn} Q_j H_j^l)$  = maximum surplus power.

The network resilience can also be viewed as equivalent to the resilience index with surplus power at each node  $j$  given a weight of  $C_j$  based on the uniformity in diameter of pipes connected to it.

Theoretically, the value of network resilience may vary between 0 and 1. However, for real systems it never attains a value of 1. It must be noted that forcing diameters of all pipes of a network to be the same need not always provide a Pareto-optimal solution in cost- $I_n$  space, as  $I_n$  is a measure of combined effect of surplus power and nodal uniformity.

## Multiobjective Genetic Algorithms

Many real world engineering design problems involve simultaneous optimization of multiple objectives. In single objective optimization, the goal is to find the best design solution, called the global optimum. Conversely, in a multicriterion optimization with conflicting objectives, there is no single optimal solution. The interaction among different objectives gives rise to a set of compromised solutions, largely known as the Pareto-optimal solutions. Since none of these Pareto-optimal solutions can be identified as better than others without any further consideration, the goal in a multicriterion optimization is to find as many Pareto-optimal solutions as possible. Once such solutions are found, it usually requires higher-level decision making with other considerations to choose one of them for implementation.

In dealing with multicriterion optimization problems, classical search and optimization methods are not efficient, simply because (1) most of them cannot find multiple solutions in a single run, thereby requiring them to be applied as many times as the number of desired Pareto-optimal solutions; (2) multiple application of these methods does not guarantee finding widely different Pareto-optimal solutions; and (3) most of them cannot efficiently handle problems with discrete variables and problems having multiple optimal solutions. On the other hand, studies based on evolutionary search algorithms, over the past few years, have shown that these methods can be efficiently used to eliminate most of the aforementioned difficulties of classical methods (Deb 2001). Because they use a population of solutions in their search, multiple Pareto-optimal solutions can, in principle, be found in one single run. The use of diversity preserving mechanisms can be added to the evolutionary search algorithms to find widely different Pareto-optimal solutions. Many multiobjective genetic algorithms such as the vector enabled GA (VEGA), multiobjective optimization GA (MOGA), niched Pareto GA, and nondominated sorting GA (NSGA), are published in the literature. In this study, NSGA, developed by Srinivas and Deb (1994), is used.

### Nondominated Sorting Genetic Algorithm

The idea behind NSGA is that a ranking method is used to emphasize current nondominated points and a niching method is used to maintain diversity in the population. NSGA can be applied either with binary coded strings or real coded strings. NSGA differs from a simple genetic algorithm only in the way the selection operator is used. The crossover and mutation operators remain as usual. Before selection is performed, first the population is ranked on the basis of an individual's nondomination level, which is found by the following procedure, and then fitness is assigned to each population member.

### Nondominated Solution

For a problem having more than one objective function, any two solutions  $x^{(1)}$  and  $x^{(2)}$  can have one of two possibilities, one dominating the other, or neither dominating the other. A solution  $x^{(1)}$  is said to dominate the other solution  $x^{(2)}$  if both the following conditions are true:

1. The solution  $x^{(1)}$  is no worse (say the operator  $\subset$  denotes worse and  $\supset$  denotes better) than  $x^{(2)}$  in all objectives, or  $f_j(x^{(1)}) \subseteq f_j(x^{(2)})$  for all  $j=1,2,\dots,M$  objectives, and
2. The solution  $x^{(1)}$  is strictly better than  $x^{(2)}$  in at least one objective, or  $f_j(x^{(1)}) \supset f_j(x^{(2)})$  for at least one  $j \in \{1,2,\dots,M\}$ .

If any of the preceding conditions is violated, the solution  $x^{(1)}$  does not dominate the solution  $x^{(2)}$ . If  $x^{(1)}$  dominates the solution  $x^{(2)}$ , then  $x^{(1)}$  is said to be a nondominated solution.

### Fitness Assignment

Consider a set of  $N$  population members, each having  $M$  ( $>1$ ) objective function values. The following procedure can be used to find the set of nondominated solutions:

- Step 0: Begin with  $i=1$ .
- Step 1: For all  $j=1,2,\dots,N$  and  $j \neq i$ , compare solutions  $x^{(i)}$  and  $x^{(j)}$  for domination using the two aforementioned conditions, for all  $M$  objectives.
- Step 2: If for any  $j$ ,  $x^{(i)}$  is dominated by  $x^{(j)}$ , mark  $x^{(i)}$  as "dominated."
- Step 3: If all solutions (i.e., when  $i=N$  is reached) in the set are considered, go to Step 4; else, increment  $i$  by one and go to Step 1.
- Step 4: All solutions that are not marked "dominated" are nondominated solutions.

All these nondominated solutions are assumed to constitute the first nondominated front in the population and assigned a large dummy fitness value (we assign fitness  $N$ ). The same fitness value is assigned to give an equal reproductive potential to all these nondominated individuals. In order to maintain diversity in the population, these nondominated solutions are then shared with their dummy fitness values. Sharing is achieved by dividing the dummy fitness value of an individual by a quantity, called the niche count, proportional to the number of individuals around it. This procedure causes multiple optimal points to coexist in the population. The worst shared fitness value in the solutions of the first nondominated front is noted for further use. After sharing, these nondominated individuals are ignored temporarily to process the rest of the population members. The above step-by-step procedure is used to find the second level of nondominated solutions in the population. Once they are identified, a dummy fitness value, which is a little smaller than the worst shared fitness value observed in solutions of the first nondominated set, is assigned. Thereafter, the sharing procedure is performed among the solutions of the second nondomination level and shared fitness values are found as before. This process is continued until all population members are assigned a shared fitness value. The population is then reproduced with the shared fitness values. In this study, a real coded NSGA with tournament selection, arithmetic crossover, and Gaussian mutation, which are explained in the following sections, is used.

### Sharing Procedure

Given a set of  $n_k$  solutions in the  $k$ th nondominated front, each having a dummy fitness value  $f'_k$ , the sharing procedure is performed in the following way for each solution  $i=1,2,\dots,n_k$ :

- Step 1: Compute a normalized Euclidean distance measure with another solution  $j$  in the  $k$ th nondominated front, as follows:

$$d_{ij} = \sqrt{\sum_{p=1}^P \left( \frac{x_p^{(i)} - x_p^{(j)}}{x_p^u - x_p^l} \right)^2} \quad (18)$$

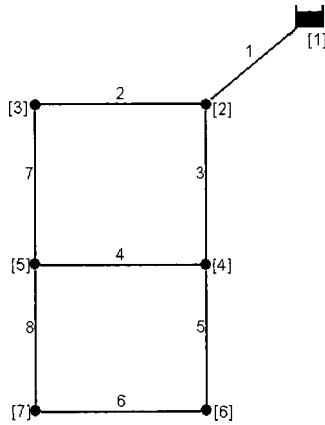


Fig. 1. Two-loop network

where  $P$  = number of decision variables in the problem. The parameters  $x_p^u$  and  $x_p^l$  are the upper and lower bounds of variable  $x_p$ .

- Step 2: This distance  $d_{ij}$  is compared with a prespecified parameter  $\sigma_{share}$  = following sharing function value is computed:

$$Sh(d_{ij}) = \begin{cases} 1 - \left( \frac{d_{ij}}{\sigma_{share}} \right)^2 & \text{if } d_{ij} \leq \sigma_{share} \\ 0 & \text{otherwise} \end{cases} \quad (19)$$

- Step 3: Increment  $j$ . If  $j \leq n_k$ , go to Step 1. If  $j > n_k$ , calculate niche count for the  $i$ th solution as follows:

$$m_i = \sum_{j=1}^{n_k} Sh(d_{ij}) \quad (20)$$

- Step 4: Degrade the dummy fitness  $f'_k$  of the  $i$ th solution in the  $k$ th nondomination front to calculate the shared fitness,  $f_i$  as follows:

$$f_i = \frac{f'_k}{m_i} \quad (21)$$

This procedure is continued for all  $i = 1, 2, \dots, n_k$  and a corresponding  $f_i$  is found. Thereafter, the smallest value  $f_k^{\min}$  of all  $f_i$  in the  $k$ th nondominated front is found for further processing. The dummy fitness of the next nondominated front is assigned to be  $f'_{k+1} = f_k^{\min} - \epsilon_k$ , where  $\epsilon_k$  is a small positive number.

The preceding sharing procedure requires a prespecified parameter  $\sigma_{share}$ , which can be calculated as follows:

$$\sigma_{share} \approx \frac{0.5}{\sqrt{p}} \quad (22)$$

Although the calculation of  $\sigma_{share}$  depends on  $q$ , the use of the above equation with  $q \approx 10$  works well. Moreover, the performance of NSGA is not very sensitive to this parameter near  $\sigma_{share}$  values calculated using  $q \approx 10$  (Srinivas and Deb 1994).

### Arithmetic Crossover

If we assume  $x^{(1)} = (x_1^1, x_2^1, \dots, x_{nd}^1)$  and  $x^{(2)} = (x_1^2, x_2^2, \dots, x_{nd}^2)$  are two parents selected for crossover, then two offspring are generated as follows:

$$y^{(k)} = (y_1^k, y_2^k, \dots, y_{nd}^k) \quad k = 1, 2 \quad (23)$$

where  $y_i^1 = \lambda x_i^1 + (1 - \lambda)x_i^2$ ;  $y_i^2 = (1 - \lambda)x_i^1 + \lambda x_i^2$ ; and  $\lambda = \text{constant}$  ( $0 \leq \lambda \leq 1$ ). This crossover operator was found to

Table 1. Pipe Cost Data for Two-Loop Network

Diameter (mm)	Cost (\$/m)
25.4	2
50.8	5
76.2	8
101.6	11
152.4	16
203.2	23
254.0	32
304.8	50
355.6	60
406.4	90
457.2	130
508.0	170
558.8	300
609.6	550

give good results for water distribution network optimization with  $\lambda = 0.75$  (Vairavamoorthy and Ali 2000). The same is used in this study also.

### Gaussian Mutation

If  $y(k)$  is an offspring and  $y_i^k$  is a gene randomly selected for mutation, then the gene obtained after Gaussian mutation is as follows:

$$z_i^k = y_i^k + N(0, \sigma) \quad (24)$$

where  $N(0, \sigma)$  = random Gaussian number with mean zero and standard deviation  $\sigma = f(y_i^u)$ , where  $y_i^u$  is the maximum value of the gene. Here the value of  $\sigma = 0.1 \times y_i^u$  is used. With this scheme applied, if new gene values exceed their range at either end, the values are adjusted to take limiting values.

### Constraint Handling

Although the nodal mass balance and loop energy balance equations [Eq. (3)] are satisfied externally by using a hydraulic network solver, the other constraints [Eq. (4)] must be satisfied within the framework of a GA. In the previous GA applications to water distribution network optimization, many improvements were suggested for constraint handling (Dandy et al. 1996; Savic and Walters 1997; Vairavamoorthy and Ali 2000). However, these methods are not elegant in the sense that they all require a penalty coefficient. Identifying a penalty coefficient is a difficult task, and it may change from problem to problem. The penalty coefficient must take a value that will not allow the best infeasible solution to be better than any feasible solution in the population (Simpson et al. 1994; Savic and Walters 1997). In this study a method of constraint handling that does not require a penalty coefficient to be specified and is applicable to water distribution network is developed. The method was first introduced by Deb and Agrawal (1999) and is modified here to fit for water distribution network optimization.

A solution  $x^{(i)}$  is constraint-dominating a solution  $x^{(j)}$ , if any of the following are true:

1. Solution  $x^{(i)}$  is feasible and solution  $x^{(j)}$  is infeasible,
2. Solution  $x^{(i)}$  and  $x^{(j)}$  are both infeasible, but  $x^{(i)}$  has a smaller constraint violation, or

**Table 2.** Node Data for Two-Loop Network

Node	Minimum head (m)	Demand (m <sup>3</sup> /h)
1	210.0	-1,120.0
2	180.0	100.0
3	190.0	100.0
4	185.0	120.0
5	180.0	270.0
6	195.0	330.0
7	190.0	200.0

3. Solution  $x^{(i)}$  and  $x^{(j)}$  are feasible and solution  $i$  dominates solution  $j$ .

This way, feasible solutions are constraint-dominated to any infeasible solution and two infeasible solutions are compared based on their constraint violations only. However, when two feasible solutions are compared, they are checked on their domination level (fitness value). The constraint violation for any solution can be calculated using a failure index as

$$I_j = \frac{\sum_{j=1}^{nn} e_j}{\sum_{k=1}^{nr} Q_k H_k + \sum_{i=1}^{npu} (P_i / \gamma)} \quad (25)$$

where

$$e_j = \begin{cases} 0 & \text{when } H_j \geq H_j^l \\ Q_j(H_j^l - H_j) & \text{otherwise} \end{cases}$$

The preceding constraint handling procedure does not require any penalty coefficient and a feasible solution always has more priority than any infeasible solution.

## Comparison of Reliability Measures

The efficacy of a reliability measure can be accessed by its ability to prioritize the solutions according to the benefit accrued from them. This aspect is investigated using the simple network shown in Fig. 1, solved first by Alperovits and Shamir (1977) and later by many investigators. It is a two-loop network with seven nodes and eight pipes, each having a length of 1,000 m. Pipe cost data and node data are given in Tables 1 and 2, respectively. The discrete search space for this example consists of  $14^8 (= 1.48 \times 10^9)$  solutions. Complete enumeration of these solutions with single loading took about 30 h of CPU time using a Pentium-III processor. The following loading conditions were used in the analysis:

- Design criterion 1 (DC1): Satisfy baseline demands and minimum required heads as defined in Table 2.
- Design criterion 2 (DC2): Satisfy baseline demands and minimum required heads as defined in Table 2 under the single pipe outage scenario (except for pipe 1).

### Case 1

Initially the ability of the reliability measure to prioritize the solutions according to benefit accrued from them was investigated. To this end, the two-loop network was completely enumerated to find the solutions that satisfy DC1. Table 3 gives the best five feasible solutions for each of the aforementioned reliability measures. Solutions 1–5 (Table 3) are the maximum  $I_m$  solutions. It can be observed that, in all these five solutions, pipes 4 and 6 have smallest diameters. Also, it was observed that most of the solutions along the Pareto-front in cost- $I_m$  space have small diameters for pipes 4 and 6. This aspect discourages the use of  $I_m$

**Table 3.** Two-Loop Network: Best Five Solutions for Each Reliability Measure

Number	Diameter of pipe (mm)								Cost $\times 10^6$ (\$)	$I_n$	$I_r$	$I_m$ (m)	$I_t$ (m)
	1	2	3	4	5	6	7	8					
Maximize $I_m$													
1	609.6	609.6	609.6	25.4	609.6	25.4	609.6	609.6	3.304	0.6223	0.9002	12.8559	127.0719
2	609.6	609.6	609.6	25.4	609.6	25.4	609.6	558.8	3.054	0.6141	0.8999	12.8559	127.0360
3	609.6	609.6	609.6	25.4	609.6	25.4	609.6	508.0	2.924	0.6059	0.8994	12.8559	126.9743
4	609.6	609.6	609.6	25.4	609.6	25.4	558.8	609.6	3.054	0.6091	0.8969	12.8558	126.7211
5	609.6	609.6	609.6	25.4	609.6	25.4	609.6	457.2	2.884	0.5976	0.8986	12.8558	126.8630
Maximize $I_t$													
6	609.6	609.6	609.6	609.6	609.6	203.2	609.6	609.6	3.873	0.8007	0.9038	12.6999	127.5184
7	609.6	609.6	609.6	609.6	609.6	152.4	609.6	609.6	3.866	0.7878	0.9037	12.6944	127.5183
8	609.6	609.6	609.6	609.6	609.6	254.0	609.6	609.6	3.882	0.8136	0.9037	12.7065	127.5182
9	609.6	609.6	609.6	609.6	609.6	101.6	609.6	609.6	3.861	0.7749	0.9036	12.6907	127.5181
10	609.6	609.6	609.6	609.6	609.6	76.2	609.6	609.6	3.858	0.7685	0.9036	12.6897	127.5180
Maximize $I_r$													
11	609.6	609.6	609.6	609.6	609.6	609.6	609.6	609.6	4.400	0.9038	0.9038	12.7292	127.5159
12	609.6	609.6	609.6	609.6	609.6	558.8	609.6	609.6	4.150	0.8909	0.9038	12.7284	127.5160
13	609.6	609.6	609.6	609.6	609.6	508.0	609.6	609.6	4.020	0.8780	0.9038	12.7273	127.5162
14	609.6	609.6	609.6	609.6	609.6	457.2	609.6	609.6	3.980	0.8651	0.9037	12.7254	127.5165
15	609.6	609.6	609.6	609.6	609.6	406.4	609.6	609.6	3.940	0.8523	0.9037	12.7226	127.5169
Maximize $I_n$													
16	609.6	609.6	609.6	609.6	609.6	609.6	609.6	609.6	4.400	0.9038	0.9038	12.7292	127.5159
17	609.6	609.6	609.6	609.6	558.8	558.8	609.6	609.6	3.900	0.8941	0.9030	12.6935	127.4472
18	609.6	609.6	609.6	609.6	558.8	609.6	609.6	609.6	4.150	0.8931	0.9030	12.6956	127.4473
19	609.6	609.6	558.8	609.6	609.6	609.6	609.6	609.6	4.150	0.8927	0.8989	12.6011	126.9401
20	609.6	609.6	609.6	558.8	609.6	609.6	609.6	609.6	4.150	0.8923	0.9037	12.7277	127.5046

**Table 4.** Two-Loop Network: Solutions with Cost \$870,000

Diameter of pipe (mm)								$I_n$	$I_r$	$I_m$ (m)	$I_t$ (m)
1	2	3	4	5	6	7	8				
Feasible solutions that satisfy DC2											
508.0	508.0	457.2	406.4	355.6	355.6	457.2	355.6	0.67	0.72	7.56	104.60
508.0	508.0	457.2	355.6	355.6	355.6	457.2	406.4	0.67	0.72	7.91	105.29
508.0	457.2	457.2	406.4	355.6	355.6	508.0	355.6	0.65	0.71	7.47	103.72
508.0	457.2	457.2	355.6	355.6	355.6	508.0	406.4	0.64	0.72	7.82	104.34
Maximum $I_m$ , $I_t$ , $I_r$ , and $I_n$ solutions, respectively, that satisfy DC1											
558.8	457.2	508.0	25.4	457.2	152.4	406.4	254.0	0.54	0.76	10.63	110.42
558.8	457.2	508.0	254.0	406.4	76.2	406.4	304.8	0.59	0.78	10.04	112.60
558.8	457.2	508.0	254.0	406.4	76.2	406.4	304.8	0.59	0.78	10.04	112.60
558.8	406.4	508.0	355.6	406.4	304.8	355.6	304.8	0.71	0.77	9.13	112.55

as a reliability measure in multiobjective optimization. Solutions 6–10 (Table 3) are the maximum total head ( $I_t$ ) solutions. It can be observed that these solutions have small diameters for pipe 6. Also, it was observed that most of the solutions along the Pareto-front in cost- $I_t$  space have small diameters for pipes 4 and/or 6. Another important observation is that both the maximum  $I_m$  solution and the maximum  $I_t$  solution are different from the maximum cost solution:  $D=[609.6, 609.6, 609.6, 609.6, 609.6, 609.6, 609.6]$  mm.

Solutions 11–15 (Table 3) are the maximum resilience index ( $I_r$ ) solutions. Comparatively,  $I_r$  appears to be a better reliability measure, as the maximum cost solution has the maximum resilience index value. However, it was observed that some of the solutions along the Pareto-front in cost- $I_r$  space have small diameters for pipes 4 and/or 6. This can be understood as the solution  $D=[609.6, 609.6, 609.6, 609.6, 609.6, 203.2, 609.6, 609.6]$  mm has a better resilience index value ( $I_r=0.903691$ ) than the solution  $D=[609.6, 609.6, 609.6, 558.8, 609.6, 609.6, 609.6, 609.6]$  mm, which has a resilience index value of 0.903686. In contrast, the designs 16–20 (Table 3)—maximum network resilience solutions—are providing both increased capacity and redundancy. It can be observed from these solutions that the prioritization of designs is performed based on both output power and redundancy.

### Case 2

The ability of a reliability measure to represent the effect of redundancy was then investigated. For this purpose, the two-loop network was solved to satisfy DC2. Through complete enumeration of the discrete search space it was found that there are four

least-cost (\$870,000) solutions that satisfy DC2. These solutions are presented in Table 4. It was also found from complete enumeration of the two-loop network that there are 32,174 solutions that satisfy DC1 and have a network cost of \$870,000. Maximum  $I_m$ ,  $I_t$ ,  $I_r$ , and  $I_n$  solutions, selected from the aforementioned 32,174 solutions, are also presented in Table 4. In a multiobjective optimization with a single loading, these solutions would have been selected as nondominated solutions. It can be observed from Table 4 that the solutions of  $I_m$ ,  $I_t$ , and  $I_r$  have small diameters for pipes 4 and/or 6. Conversely, the  $I_n$  solution has loops with practicable diameters. None of these solutions match with the four feasible solutions (Table 4) for the pipe breakage case. However, the solution provided by  $I_n$  is far superior as compared with the solutions of the other reliability measures. Similar results (Table 5) were also observed with various levels of redundancy such as outage of any pipe in loop 2 (i.e., outage of any pipe among pipes 4, 5, 6, and 8). It must be noted that maximization of the network resilience improves the reliability of network solutions but does not guarantee the delivery of water at different nodes under a failure condition.

### Application Model

The proposed multiobjective algorithm was applied to two example problems. The first example is a two-loop network used in the previous section and the second is a trunk network of Hanoi, Vietnam. The results obtained were then used to compare reliability measures. Sensitivity of the GA parameter  $\sigma_{share}$  on the Pareto-front was also investigated using these examples.

**Table 5.** Two-Loop Network: Solutions with Cost \$710,000

Diameter of pipe (in.)								$I_n$	$I_r$	$I_m$ (m)	$I_t$ (m)
1	2	3	4	5	6	7	8				
Feasible solutions											
508.0	406.4	406.4	406.4	355.6	355.6	355.6	406.4	0.60	0.64	5.68	95.32
508.0	406.4	406.4	355.6	355.6	406.4	355.6	406.4	0.59	0.63	5.64	94.97
508.0	406.4	406.4	355.6	355.6	355.6	406.4	406.4	0.61	0.65	6.12	96.66
508.0	355.6	406.4	406.4	355.6	406.4	355.6	406.4	0.58	0.61	5.13	91.83
508.0	355.6	406.4	355.6	355.6	406.4	406.4	406.4	0.57	0.62	5.37	92.31
Maximum $I_m$ , $I_t$ , $I_r$ , and $I_n$ solutions, respectively, that satisfy DC1											
508.0	457.2	457.2	50.8	457.2	50.8	406.4	304.8	0.47	0.68	8.97	100.12
558.8	355.6	457.2	76.2	406.4	355.6	355.6	25.4	0.45	0.69	6.96	102.93
508.0	457.2	457.2	254.0	406.4	76.2	406.4	355.6	0.55	0.70	7.91	102.72
508.0	355.6	508.0	355.6	406.4	304.8	355.6	304.8	0.65	0.69	7.41	101.81

**Table 6.** Two-Loop Network: Some Solutions Obtained Using NSGA

Pipe	Maximize $I_n$ (diameter in mm)				Maximize $I_r$ (diameter in mm)			
	1	2	3	4	1	2	3	4
1	457.2	457.2	457.2	457.2	457.2	457.2	457.2	508.0
2	355.6	355.6	304.8	355.6	254.0	355.6	406.4	355.6
3	355.6	355.6	406.4	406.4	406.4	355.6	355.6	355.6
4	50.8	203.2	254.0	254.0	101.6	25.4	25.4	101.6
5	355.6	355.6	355.6	355.6	406.4	355.6	355.6	304.8
6	152.4	50.8	152.4	152.4	254.0	152.4	25.4	50.8
7	355.6	355.6	254.0	254.0	254.0	355.6	355.6	355.6
8	254.0	254.0	254.0	254.0	25.4	254.0	254.0	254.0
Cost (\$)	423,000	430,000	442,000	452,000	419,000	420,000	436,000	448,000
$I_r$	0.3451	0.3508	0.3339	0.3675	0.2103	0.3444	0.3875	0.4125
$I_n$	0.2544	0.2887	0.3063	0.3370	0.1535	0.2488	0.2763	0.3039

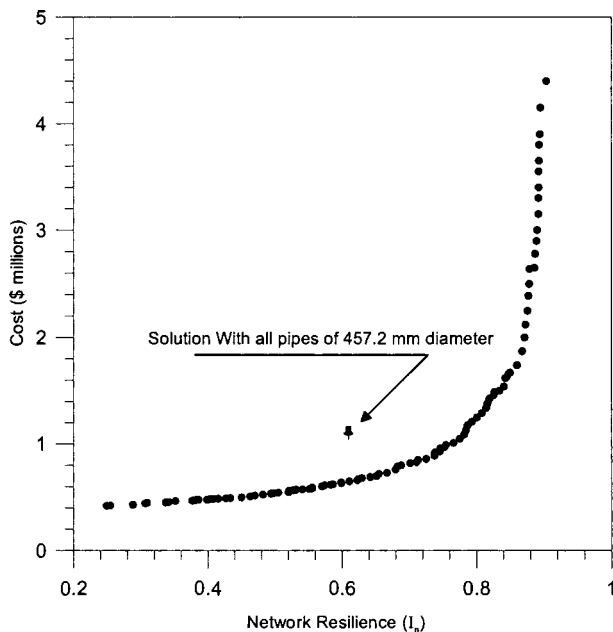
**Example 1**

The two-loop network (Fig. 1) is a typical network, as it contains many alternative solutions with the same network cost. There are eight decision variables in this example; i.e., each pipe can take any of the 14 discrete diameters listed in Table 1. Initially, the model was applied with the objectives of minimizing cost and maximizing network resilience. The GA parameters used for this run were population size=100; probability of crossover=1.0; probability of mutation=0.05;  $\sigma_{share}$ =0.375; and number of generations=1,000. Some of the designs obtained using this approach along with their cost, network resilience, and resilience index are presented in Table 6. These designs not only have increased surplus power at the nodes, but also have loops with practicable diameters. The Pareto-front obtained is shown in Fig. 2. As mentioned before, forcing the diameters of all pipes of a network to be the same does not always lead to a Pareto-optimal solution. For example, the solution  $D=[457.2, 457.2, 457.2, 457.2, 457.2, 457.2, 457.2, 457.2]$  mm has the same diameter for all pipes. The plotting position of this solution in cost- $I_n$  space is as shown in Fig. 2. From this it can be realized that this solution

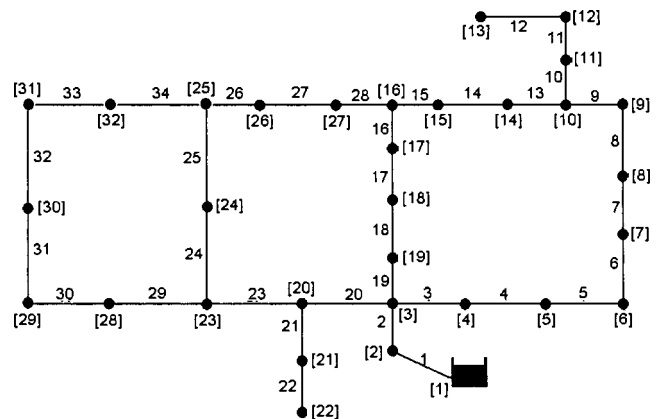
is not a Pareto-optimal solution. In order to realize the effect of  $C_j$  on reliability measure, the two-loop network was solved with the objectives of minimizing cost and maximizing resilience index. The GA parameters used were the same as before. Some of the designs obtained using this approach along with their cost, network resilience, and resilience index are presented in Table 6. These results further substantiate that the use of resilience index as a reliability measure in the model has improved surplus power at the nodes, but it could not eliminate impracticable loops. In view of the preceding observations, it is suggested to use network resilience as a measure of reliability in multiobjective analysis.

**Example 2**

The present method was also applied to another benchmark problem, the Hanoi trunk network. This example was used to investigate the sensitivity of  $\sigma_{share}$  in the derivation of the Pareto-front. The layout of the network is shown in Fig. 3 (Fujiwara and Khang 1990). This network consists of 32 nodes and 34 pipes and is supplied by a fixed grade source at an elevation of 100 m. The minimum required head at the junction nodes is specified to be 30 m. The set of commercially available diameters (in inches) is  $A=[304.8, 406.4, 508.0, 609.6, 762.0, 1,016.0]$  mm, and their corresponding cost per unit length is calculated using the equation  $1.1 \times D^{1.5}$ . For this problem, the number of decision variables was 34 and the GA parameters used were population size=200; probability of crossover=1.0; probability of mutation=0.01;  $\sigma_{share}$ =0.467 (i.e.,  $q \approx 10$ ); and number of generations=10,000. This



**Fig. 2.** Two-loop network: Pareto-front in cost- $I_n$  space



**Fig. 3.** Hanoi network



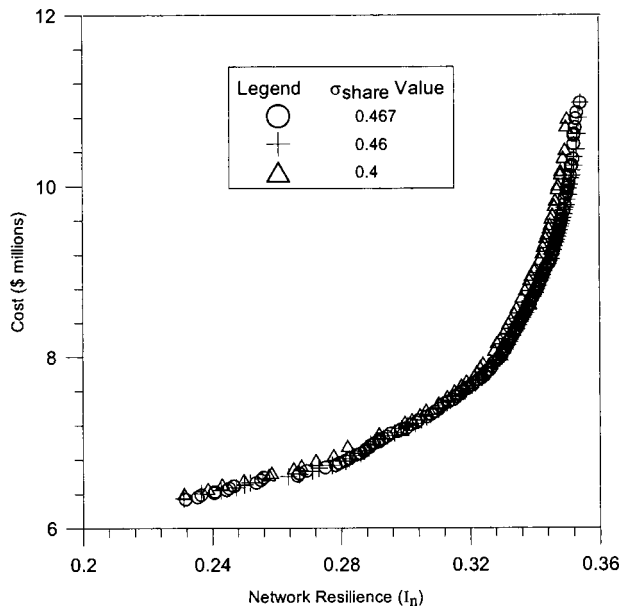


Fig. 4. Hanoi network: Pareto-front in cost- $I_n$  space

example further substantiates the observations made for the two-loop network. The Pareto-front in cost- $I_n$  space is as shown in Fig. 4. Cost and network resilience values for some of the solutions along the Pareto-front are given in Table 7. Using the same set of GA parameters as before and different values of  $\sigma_{share}$ , the model was applied. The Pareto-fronts obtained for  $\sigma_{share}=0.46$  and 0.4 (i.e.,  $q \approx 17$  and 1,972, respectively) are also presented in Fig. 4. It can be observed from the figure that when  $\sigma_{share}$  values are calculated using Eq. (22) (i.e.,  $q \approx 10$ ), the Pareto-front contained many solutions distributed along the front. Though the Pareto-front obtained with  $\sigma_{share}=0.4$  converged towards the Pareto-front, the number of different solutions has decreased. This can be understood, as  $\sigma_{share}$  is used to get many diversified solutions. If one would like to obtain more solutions towards the lower cost region (due to limits on funding), then an additional constraint (cost  $\leq$  a predefined value) can be introduced into the

Table 7. Hanoi Network: Some Pareto-Optimal Solutions in Cost- $I_n$  Space

Solution number	Cost (\$)	Network resilience ( $I_n$ )	Solution number	Cost (\$)	Network resilience ( $I_n$ )
1	6,349,285.0	0.231	16	6,697,784.5	0.272
2	6,374,160.0	0.234	17	6,701,748.5	0.273
3	6,406,231.0	0.237	18	6,731,132.0	0.276
4	6,430,537.5	0.242	19	6,736,188.5	0.277
5	6,444,537.5	0.243	20	6,768,259.5	0.278
6	6,457,077.5	0.244	21	6,783,057.5	0.281
7	6,476,932.5	0.247	22	6,795,963.0	0.282
8	6,509,003.5	0.249	23	6,811,428.0	0.283
9	6,535,294.0	0.252	24	6,825,057.5	0.283
10	6,561,047.5	0.255	25	6,847,828.0	0.284
11	6,578,748.0	0.256	26	6,873,552.0	0.286
12	6,604,863.5	0.257	27	6,900,152.0	0.287
13	6,631,273.5	0.267	28	6,901,996.5	0.287
14	6,660,657.0	0.269	29	6,934,696.0	0.288
15	6,665,713.5	0.271	30	6,938,396.5	0.289

model as explained in Halhal et al. (1997). Also, it can be observed from Figs. 2 and 4 that there are few solutions (gaps) near the lower end of the reliability measure. One of the reasons for this is that there are more infeasible solutions in this range and thereby gap(s) in the Pareto-front. That is, the boundary between feasible and infeasible solutions is not a smooth rising curve.

## Conclusions

Most of the water distribution network optimization models have considered network cost as a sole objective. This may be due to the computational complexity involved in considering the other legitimate objectives such as reliability, redundancy, and water quality. This paper describes the application of a multiobjective genetic algorithm model to the design of a water distribution network. The objectives considered in this study are minimization of network cost and maximization of a reliability measure. The reliability measure used is network resilience, which is a measure of both the nodal surplus power and the uniformity in diameters connected to that node. Increase in the value of network resilience improves the reliability of a network under failure conditions. Genetic algorithms were observed to be weak at handling constraints. Therefore, a better constraint handling technique that does not require a penalty coefficient and is applicable to water distribution networks is presented. This technique ensures that a feasible solution is better than any infeasible solution in the population. Application of the model to the example problems revealed the superiority of the network resilience based approach. The method produces a set of Pareto-optimal solutions in the search space of cost and network resilience. The designer can use high level decision-making tools to select an appropriate design from the Pareto set.

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