Decision under uncertainty: why not expected utilities?

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AOS04 master courses

A quick reminder of expected utility

- Set of states $\mathscr{X}$
- Probability masses $p(x)$ for each state
- Act/decision a maps each state to a utility

$$a : \mathscr{X} \to \mathbb{R}$$

- Expected utility of a

$$\mathbb{E}(a) = \sum_x p(x)a(x)$$

Can be justified in various ways (see previous lectures)

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Outline

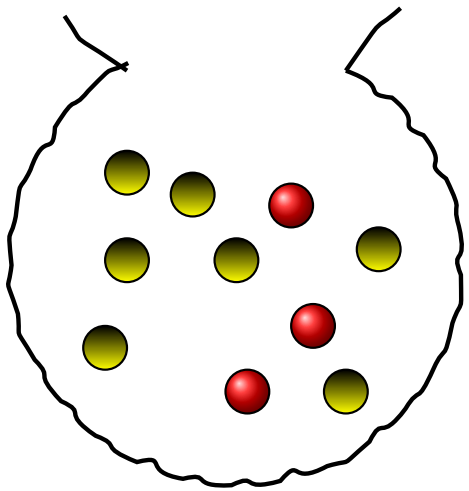
- First, some games
- Discussing the results

Experimental protocol usually followed

- Half the room goes out
- The rest pick a choice
- We exchange (inside goes outside, and vice-versa)

Urns and balls: case 1

9 balls, 3 are reds, 6 remaining are either yellow or black



What would you choose between A and B?

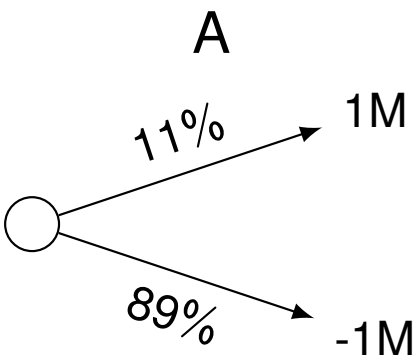
A

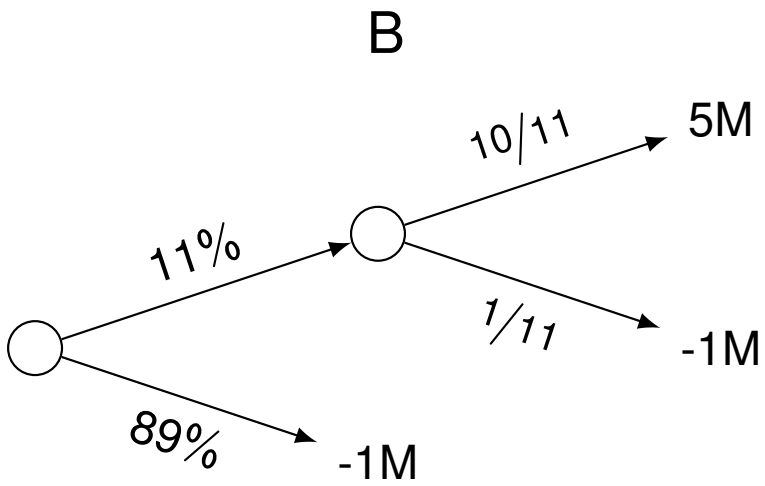
R(ed)	B(lack)	Y(ellow)
100 $	0 $	0$

B

R(ed)	B(lack)	Y(ellow)
0 $	100 $	0$

Lotteries: case 1

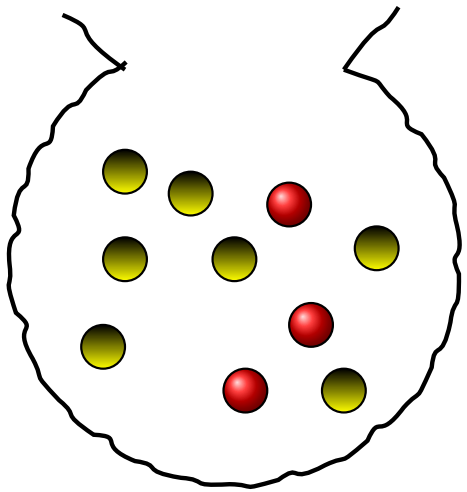




Which of the two cases would you choose?

Urns and balls: case 2

9 balls, 3 are reds, 6 remaining are either yellow or black



What would you choose between C and D?

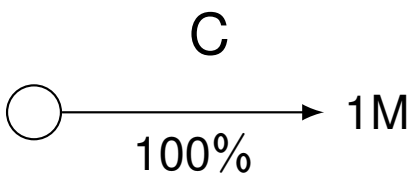
C

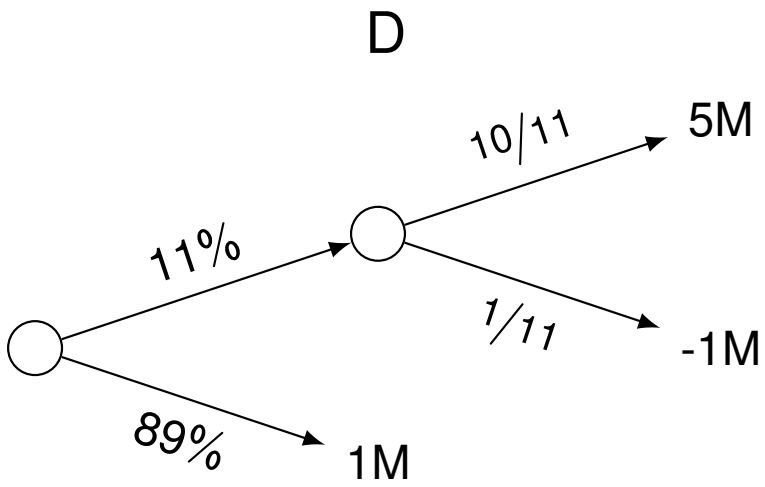
R(ed)	B(lack)	Y(ellow)
100 $	0 $	100$

D

R(ed)	B(lack)	Y(ellow)
0 $	100 $	100$

Lotteries: case 2





Which of the two cases would you choose?

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Outline

- First, some games
- Discussing the results

The urn games [2]

A

R(ed)	B(lack)	Y(ellow)
100 $	0 $	0$

B

R(ed)	B(lack)	Y(ellow)
0 $	100 $	0$

C

R(ed)	B(lack)	Y(ellow)
100 $	0 $	100$

D

R(ed)	B(lack)	Y(ellow)
0 $	100 $	100$

- Most people prefer *A* to *B*, but *D* to *C*
- Show that it conflicts with expected utility
- What do you think is the reason?

The urn games [2]

A

R(ed)	B(lack)	Y(ellow)
100 $	0 $	0$

B

R(ed)	B(lack)	Y(ellow)
0 $	100 $	0$

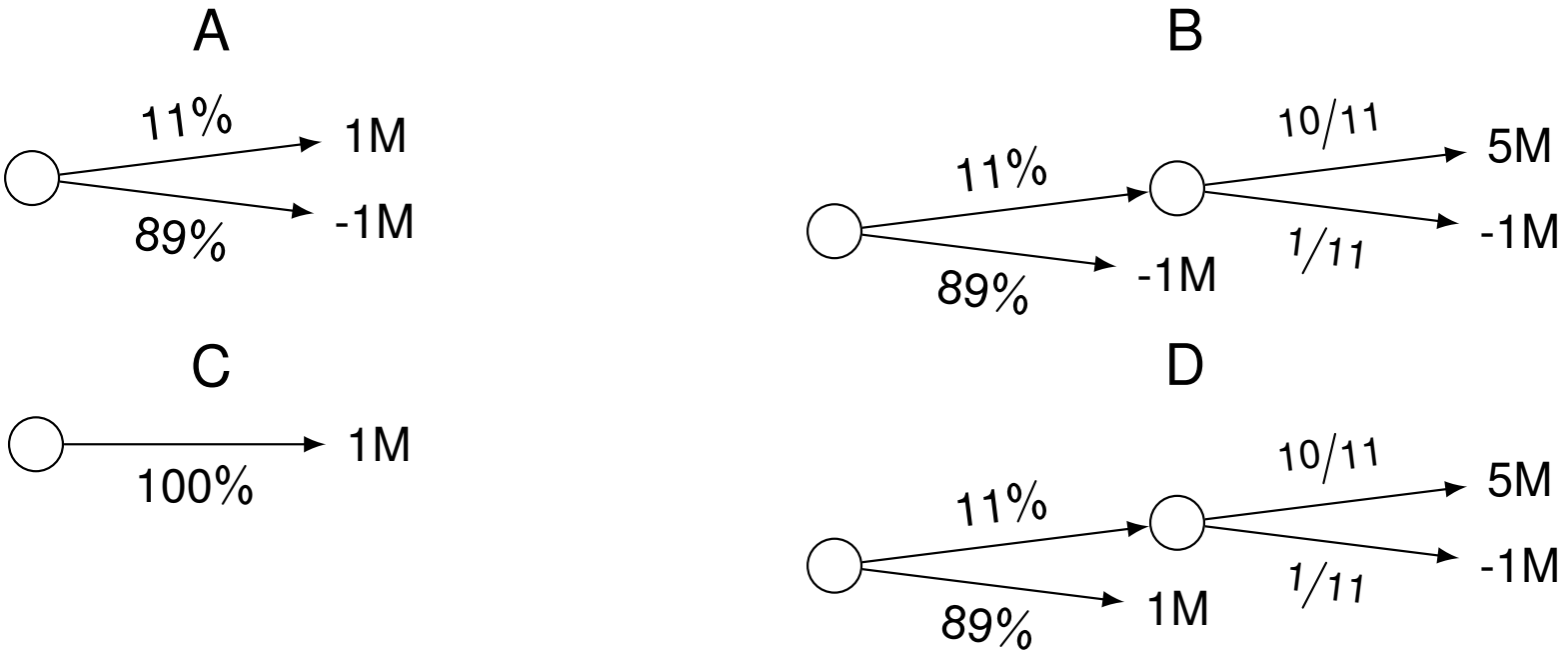
C

R(ed)	B(lack)	Y(ellow)
100 $	0 $	100$

D

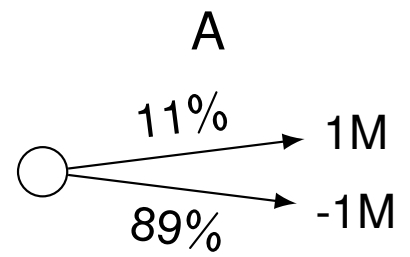
R(ed)	B(lack)	Y(ellow)
0 $	100 $	100$

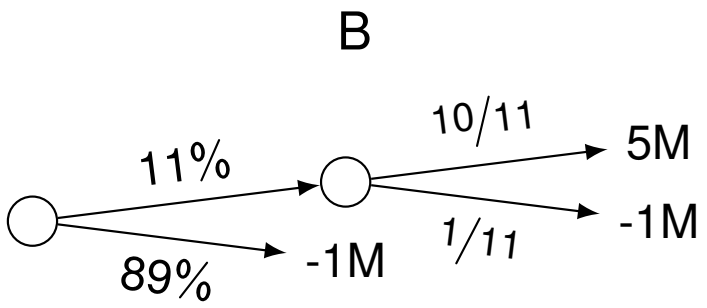
The lotteries [1]

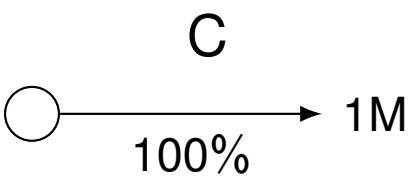


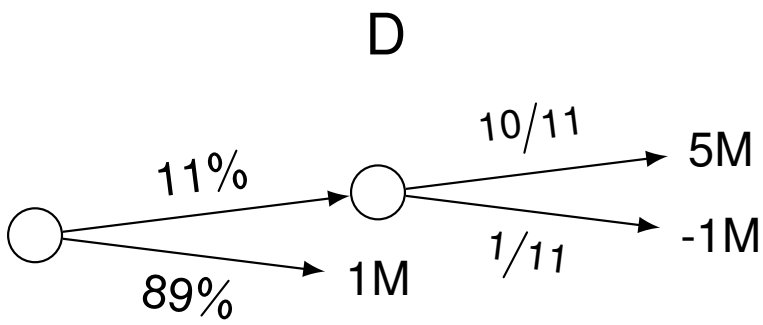
- Most people prefer *B* to *A*, but *C* to *D*
- Show that it conflicts with expected utility
- What do you think is the reason?

The lotteries [1]









Main reasons put forward

- In **both** cases, the fact that the decision maker is adverse to risk, or perceive situations differently[1] is put forward.
- In **both** cases, the sure-thing principle is violated
- In **the urn case**, this most probably come from **a lack of** knowledge of the probabilities, as we do not know them fully
- In **the lottery case**, probabilities are fully specified, but perception of small/high probabilities seem to be the problem

[1] While theory tells they should be the same

How to address the problem?

- No problem, the theory is about how people SHOULD act, not how they actually act → a no answer to an existing concern
- The problems/paradoxes put forward are rather artificial → so is a theory or an axiom to some extent
- We need to extend the theory/relax axioms and constraints to cope with such cases

Next lectures will address the last concern, in case you do not like the two first items.

References I

[1] M. Allais.
The so-called allais paradox and rational decisions under uncertainty.
In *Expected utility hypotheses and the Allais paradox*, pages 437–681. Springer, 1979.

[2] D. Ellsberg.
Risk, ambiguity, and the savage axioms.
The quarterly journal of economics, pages 643–669, 1961.