Decision under uncertainty: probabilities but not utilities

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Outline

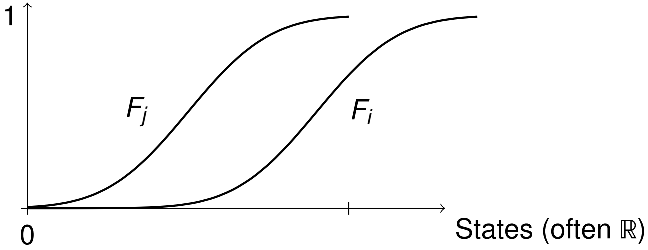
- Alternatives to utilities

Stochastic dominance

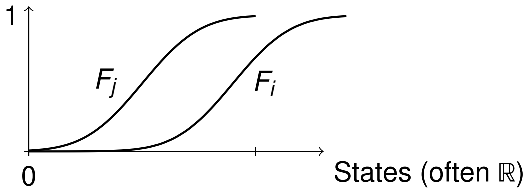
Assumptions and definitions

- States can be arranged from "worst" to "best" linearly
- For action a_i and each state x, we can estimate $F_i(x) = P(]-\infty, x])$

Then a_i stochastically dominates a_j ($a_i \succ_{SD} a_j$) if $F_i(x) \leq F_j(x)$



Why stochastic dominance?



Interests

- $a_i \succ_{SD} a_j \implies \mathbb{E}(a_i) \geq \mathbb{E}(a_j)$ for any increasing utility over $\mathscr{X}$
- Can be seen as a robust version of expected utility
- Only require to be able to rank states (no numbers necessary)

Statistical preference

Assumptions and definitions

- Utilities can be compared with each other (ordered)
- Each action/state is given a (comparable) utility grade

Then a_i is statistically preferred to a_j if $P(a_i \geq a_j) \geq 0.5$

$$p(x_3) = 0.1, p(x_2) = 0.7, p(x_1) = 0.2$$

	x_1	x_2	x_3
a_i	*high*	*high*	*low*
a_j	*low*	*very high*	*very low*
$\mathbb{1}_{a_i \geq a_j}$	1	0	1

$$P(a_i \geq a_j) = 0.1 + 0.2 = 0.3$$

$$P(a_j \geq a_i) = 0.7 \implies a_j \text{ stat. preferred to } a_j$$

Statistical preference and transitivity

Two players play the following game. Player 1 erases the spots from the faces of three fair dice and writes one number from 1, 2,. . . , 18 to each face. Each of them risks 1 €, chooses one dice, they throw the dice, and the one having the bigger number on top of his dice wins the 2 €. Since Player 1 puts the numbers to the dice it seems fair to let Player 2 choose his dice first. Of course, Player 2 tries to choose the best dice. Despite this, Player 1 wins in the long run.

How can that be?

The dice transitivity problem

Consider the following numbering of the fair Dices A, B, C

- $A = \{1, 3, 4, 15, 16, 17\}$
- $B = \{2, 10, 11, 12, 13, 14\}$
- $C = \{5, 6, 7, 8, 9, 18\}$

Assuming that dices are thrown independently, show that we have

$$P(A > B) > \frac{1}{2}, \quad P(B > C) > \frac{1}{2}, \quad P(C > A) > \frac{1}{2},$$

or, in other words, a cyclic statistical preference.

Schematic Wrap-up

Deciding under uncertainty=model of preference (utilities) + model of uncertainty + decision rule

References I

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The theory of statistical decision.
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