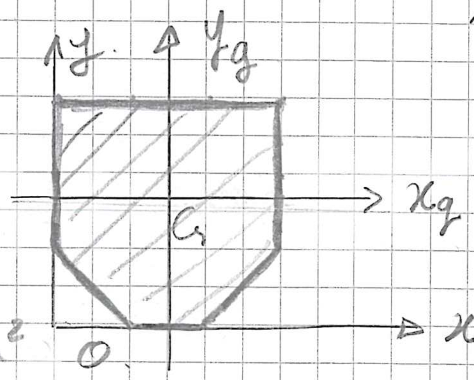


1) $G(x_G, y_G)$

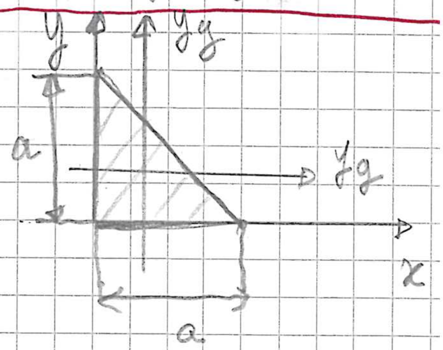
sans faire de calcul $x_G = 15$ mm

$$S = 30 \times 30 - 2\left(\frac{10 \times 10}{2}\right) = 800 \text{ mm}^2$$

$$y_G = \frac{20 \times 30 \times 20 + 10 \times 10 \times 5 + 2 \times \left(\frac{10 \times 10}{2} \times \frac{2}{3} 10\right)}{800} = 16.46 \text{ mm}$$

2) y_g étant un axe de symétrie, $I_{x_g y_g} = 0$.

Pour le calcul de I_{xg} et I_{yg}, utilise les résultats du TD pour un triangle rectangle



$$I_x = I_y = \frac{a^4}{12}$$

$$I_{xg} = I_{yg} = \frac{a^4}{36}$$

$$I_{xg} = \frac{30 \times 30^3}{12} + 30 \times 30 \times (1.46)^2 - 2\left[\frac{10^4}{36} + \frac{10 \times 10}{2} \times \left(16.46 - \frac{10}{3}\right)^2\right]$$

$$I_{xg} = 51631.94 \text{ mm}^4 = I_{min}$$

$$I_{yg} = \frac{30 \times 30^3}{12} - 2\left[\frac{10^4}{36} + \frac{10 \times 10}{2}\left(15 - \frac{10}{3}\right)^2\right] = 5333.33 \text{ mm}^4 = I_{max}$$

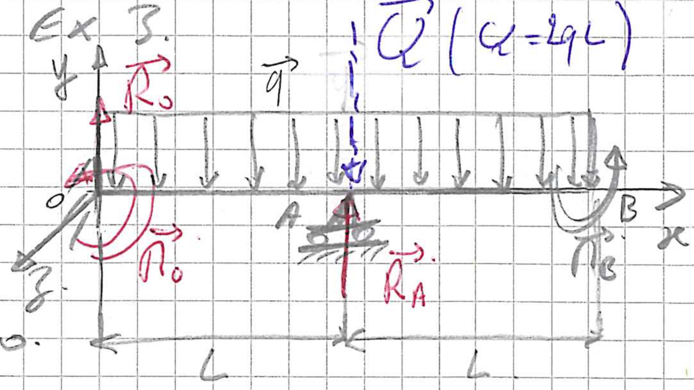
3) $I_G = I_{xg} + I_{yg} = I_{min} + I_{max} = 104965.27 \text{ mm}^4$.

4) $\theta = 0$

2/5

MQ01 Final A2019 Ex 3.

1/



2/

$\Sigma \vec{F} = 0 \Rightarrow \vec{R}_0 + \vec{R}_A + \vec{Q} = 0$

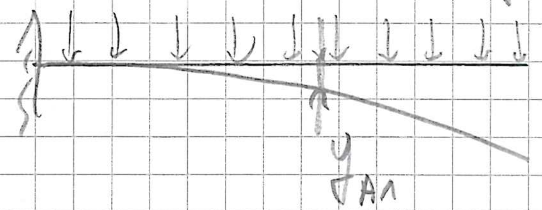
$\Rightarrow R_0 + R_A - Q = 0$

$\Sigma \vec{M}_{/0} = 0 \Rightarrow M_0 + L R_A - L Q + M_B = 0$

2 équation et 3 inconnues (R_0, M_0 et M_A). Le système est hyperstatique d'ordre 1.

3/ on utilise $y_A = 0$ (y_A désigne la flèche en A)

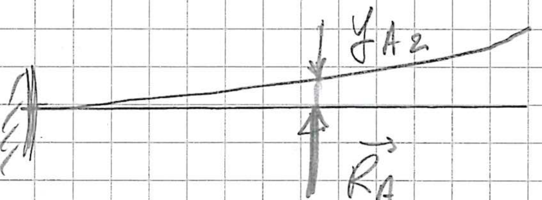
Pour exprimer y_A, on utilise la superposition



$y_1 = \frac{-q x^2}{24EI}(x^2 - 8Lx + 24L^2)$

$y_{A1} = y_1(L) = -\frac{17 q L^4}{24EI}$

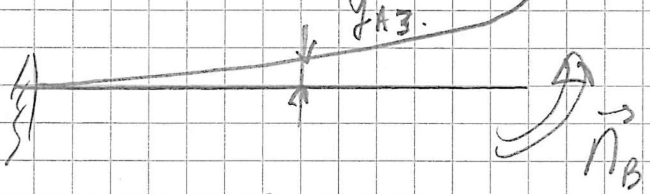
+



$y_2 = \frac{R_A x^2}{6EI}(3L - x)$

$y_{A2} = y_2(L) = \frac{R_A L^3}{3EI}$

+



$M_f = M_B$

$EI y_3'' = M_f = M_B \Rightarrow y_3'' = \frac{M_B}{EI}$

$\Rightarrow y_3' = \frac{M_B}{EI} x + C_1$

$\Rightarrow y_3 = \frac{M_B}{EI}\frac{x^2}{2} + C_1 x + C_2$

$y_3(0) = 0 \Rightarrow C_2 = 0$, $y_3'(0) = 0 \Rightarrow C_1 = 0$.

$y_3 = \frac{M_B}{EI}\frac{x^2}{2}$, $y_{A3} = y_3(L) = \frac{M_B}{EI}\frac{L^2}{2}$

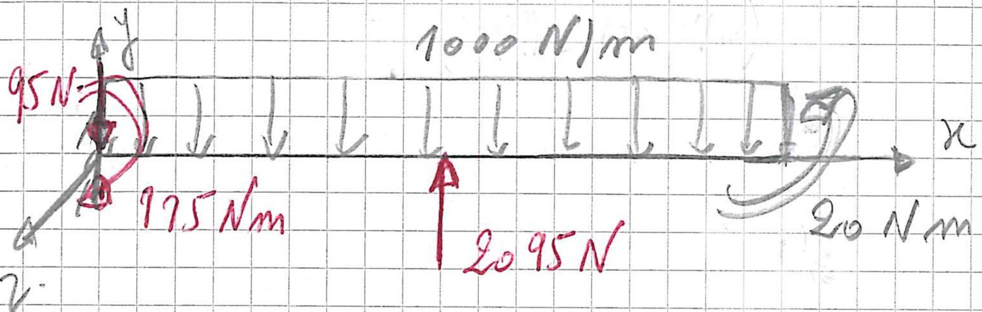
$y_A = y_{A1} + y_{A2} + y_{A3} = 0 \Rightarrow -\frac{17qL^4}{24EI} + \frac{R_A L^3}{3EI} + \frac{M_B L^2}{2EI} = 0.$

d'où $R_A = \frac{17}{8} qL - \frac{3}{2} \frac{M_B}{L}$

$R_0 = Q - R_A = 2qL - R_A = -\frac{1}{8} qL + \frac{3}{2} \frac{M_B}{L}$

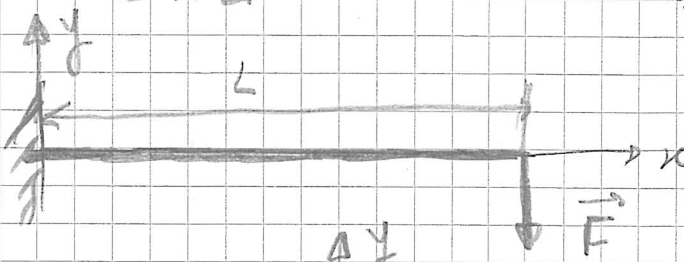
$M_0 = LQ - LR_A - M_B = 2L^2 q - LR_A - M_B = -\frac{1}{8} qL^2 + \frac{1}{2} M_B$

A.N. $R_A = 2095\ N$, $R_0 = -95\ N$, $M_0 = -115\ Nm$.



4) $\vec{R_A}$ change de sens lorsque $\frac{17}{8} qL - \frac{3}{2} \frac{M_B}{L} < 0$

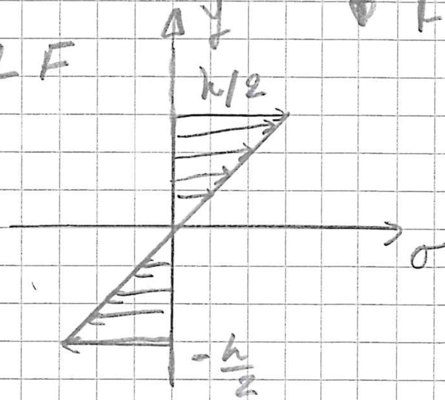
$\Rightarrow M_B > \frac{17}{12} qL^2$



1°/ $T = -F$

$M_f = -F(L-x)$

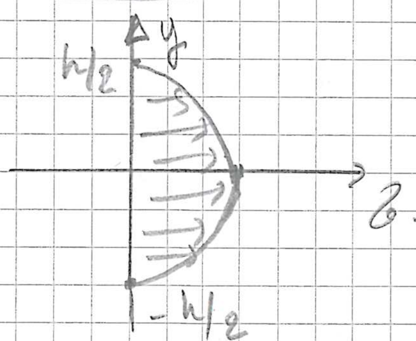
Pour $x = 0$ $T = -F$, $M_f = -LF$



2°/ $\sigma = -\dfrac{M_f}{I_{Gz}} \cdot y = \dfrac{LF}{I_{Gz}} \cdot y$

Au point B, $\sigma = \dfrac{LF}{I_{Gz}} \cdot y_J$

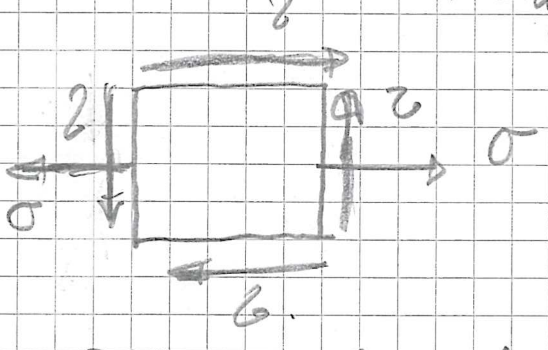
3°/ $\tau = -\dfrac{T}{I_{Gz}} \dfrac{A_{Gz}}{e} = -\dfrac{T}{I_{Gz}} \cdot \dfrac{\cancel{b}\left(\frac{h}{2}-y\right)\left(\frac{\frac{h}{2}+y}{2}\right)}{\cancel{b}}$



$\tau = \dfrac{F}{2 I_{Gz}} \left(\dfrac{h^2}{4} - y^2\right)$

Au point B $\tau = \dfrac{F}{2 I_{Gz}} \left(\dfrac{h^2}{4} - y_J^2\right)$

4°/

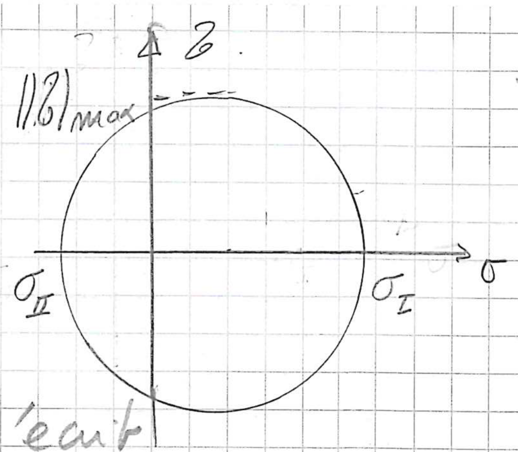


5°/ $\sigma_I = \dfrac{\sigma}{2} + \dfrac{1}{2}\sqrt{\sigma^2 + 4\tau^2}$

$\sigma_{II} = \dfrac{\sigma}{2} - \dfrac{1}{2}\sqrt{\sigma^2 + 4\tau^2}$

6)

7) $\|\tau\|_{max} = \dfrac{\sigma_I - \sigma_{II}}{2} = \dfrac{1}{2}\sqrt{\sigma^2 + 4\tau^2}$



8) la condition de résistance s'écrit

$$\|\tau\|_{max} < \frac{\sigma_a}{2}$$

$$\Rightarrow \frac{1}{2}\frac{F}{I_{Gz}}\sqrt{L^2 y_{ri}^2 + 4\frac{\left(\frac{h^2}{4} - y_{ri}^2\right)^2}{4}} < \frac{\sigma_a}{2}$$

$$\Rightarrow F < \frac{I_{Gz}\,\sigma_a}{\sqrt{L^2 y_{ri}^2 + \left(\frac{h^2}{4} - y_{ri}^2\right)^2}}$$

d'où $F < 4395\ N$

9) la condition de résistance s'écrit

$$\sigma < \sigma_a$$

$$\Rightarrow F < \frac{I_{Gz}\,\sigma_a}{L\, y_{ri}}$$

d'où $F < 4741\ N$

10) le calcul qui tient compte de τ est plus conservatif
environ 8 % d'erreur