AOS4 Homework

How to model a game with uncertainty and regularisation

Maxime Delboulle

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1 little introduction

What is the goal of the following Exercice?

You are a pirate of the Caribbean and you want to free your father from Davy Jones's ship the Flying Dutchman. To do this, you make a bet on the liar's die game: if you win, he will be free. If you lose, you'll be a slave until the end of time.

Liar's die is a game with very few steps at each turn. Each player has exactly 5 dies at the beginning. All dies are fair. At each turn, the players roll randomly the dies and cannot see the dies of the opponents. You bet on the total number of dies' spots in the game.

The bets are increasing, meaning that you have to increase the bet on the total number of dies' spot. You can suggest that your opponent is lying if its bet seems too unlikely. Each player has to reveal their dies. The player who has not respected the bet loose one die. The player who has no die is the looser.

We simplify the rules. So there is no special die's spot. There are only two players, cause you are William Turner Jr and you are facing Davy Jones.

We give an example:

INIT:

$$\Omega = \{\text{⚀}, \text{⚁}, \text{⚂}, \text{⚃}, \text{⚄}, \text{⚅}\} \qquad Turner = \{\text{⚀}, \text{⚀}, \text{⚀}, \text{⚁}, \text{⚃}, \text{⚅}\} \qquad Jones = \{\text{⚀}, \text{⚁}, \text{⚁}, \text{⚄}, \text{⚅}, \text{⚅}\}$$

The successive bets are:

1. Jones: 3 dies for 2.
2. Turner: 4 dies for 1.
3. Jones: 5 dies for 6.
4. Turner: You're a lier.
5. Jones loses one die, because there are only 3 dies for 6.

2 Answer Questions

1. How to model the bets with probability?

2. What is the probability that your opponent has 2 dies 6 knowing he has 5 dies? What is the problem?

3. Compute the previous probability knowing that the player is not lying.

4. Compute the Maxwell-Boltzmann Statistics for the previous case?

5. What is the problem with the computation of Maxwell-Boltzmann Statistics?

We assume that a pirate can bluff. So, we have to include new predictors in order to correctly guess a bet. To guess if a pirate is lying, you are using some objective and subjective predictors:

- X1: *feeling_to_lie*
- X2: *spot_has_changed*
- X3: *proba_Maxwell_Boltzmann* that has been computed earlier

At our turn, we have to predict if the opponent is lying or not. It is a classification problem. So, Y $\in$ {'lie', 'not lie'}

We give you some conditional probabilities:

- the *a priori* probability to lie is between: [0.4,0.6]
- the feeling to lie is perceived by the player. It can be $[0,\frac{1}{3}]$, $]\frac{1}{3},\frac{2}{3}]$ or $]\frac{2}{3},1]$
- X2 conditionning Y is given by this matrix:

$\mathbb{P}$(spot_has_changed|Y)	Yes	No
lie	[0.7,1]	[0,0.2]
not lie	[0,0.3]	[0.8,1]

- Maxwell-Boltzmann statistics for the beginning (opponent has 5 dies)

$\mathbb{P}$(MB_stat|Y)	1	2	3	4	5
lie	0	9%	79%	98%	99.9%
not lie	1	91%	21%	2%	0.1%

6. Are the predictors independent?

7. Considering the introductory game presentation at step 3, what would be the most likely class (NCC model)?

8. What is the main problem in the modelling?

9. Propose a way to regularise the modelling? (*Hint: we want to preserve global information and parametrises with a hyperparameter*)

10. Does it change the decision?

11. What is the difference with Dempster-Shafer theory in ISCC?

12. How many parameters and probabilities are there to estimate?

3 Conclusion

This exercise emphasises on modelling a problem that is *a priori* easy. There are many ways to model the problem. As there are lots of lyings in this game and a few steps in each turn (5 maximum), it is hard to model the right decision to take. The understanding of your opponent is decisive if you want your opponent not to play knowing your future decisions. So, you have to deal with uncertainty.

In this game, we can be such hard situations where your opponent estimates the bet correctly (or a bit underestimated). So, you have a relatively high probability to loose if you suggest he is lying. But, if you increase the bet, you have a high probability to lose. You can try to optimise your gain, but it is almost useless as you are almost sure to lose.

With this exercice, students use:

1. probability to model completely the game and see its limitations.
2. NCC (Naive Credal Classifier) and see the improvements.
3. RNCC (Regularised Naive Credal Classifier) and see the improvements.
4. compare with belief functions model.

To go further, see:

1. how to model the game with the real rule (die one is a die that can replace any other number)
2. how to compute some approximations for the Maxwell-Boltzmann statistics.
3. how to add/replace new predictors
4. test the RNCC in a real game

4 Annexe A: Maxwell-Boltzmann statistics with real game rules (pako (die number one) can represent any other die) and binomial (pako only)

```
# at least k identical dices (pako included)
get_proba_at_least_k_with_pako <- function (k, r, n, r_vect){
    proba <- 0

    if (length(r_vect) < n){
        for (r_i in 0:(r - sum(r_vect)))
        proba <- proba + get_proba_at_least_k_with_pako(k, r, n, c(r_vect, r_i))
    }
    else
        if (sum(r_vect) == r)
        if ( sum ( (k:r) %in% (r_vect[2:length(r_vect)] + r_vect[1]) ) )
            proba <- proba + MB_stat(r_vect, r, n)

        return (proba)
    }
```

See figure 2

```
# at least k identical pako dies (pako only (binomial event))
get_proba_at_least_k_only_pako <- function (k, r, n){
  proba <- 0

  for (i in 0:k-1)
    proba <- proba + dbinom(i, r, 1/n)

  return (1 - proba)
}
```

See figure 1

represent the proba of having k identical pako out of r total dices

k/r	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
1	16,7%	30,6%	42,1%	51,8%	59,8%	66,5%	72,1%	76,7%	80,6%	83,8%	86,5%	88,8%	90,7%	92,2%	93,5%	94,6%	95,5%	96,2%	96,9%	97,4%	97,8%	98,2%	98,5%	98,7%	99,0%
2	0,0%	2,8%	7,4%	13,2%	19,6%	26,3%	33,0%	39,5%	45,7%	51,5%	56,9%	61,9%	66,4%	70,4%	74,0%	77,3%	80,2%	82,7%	85,0%	87,0%	88,7%	90,2%	91,5%	92,7%	93,7%
3	0,0%	0,0%	0,5%	1,6%	3,5%	6,2%	9,6%	13,5%	17,8%	22,5%	27,3%	32,3%	37,2%	42,1%	46,8%	51,3%	55,6%	59,7%	63,6%	67,1%	70,4%	73,5%	76,3%	78,8%	81,1%
4	0,0%	0,0%	0,0%	0,1%	0,3%	0,9%	1,8%	3,1%	4,8%	7,0%	9,6%	12,5%	15,8%	19,4%	23,2%	27,1%	31,1%	35,2%	39,3%	43,3%	47,3%	51,2%	54,9%	58,4%	61,8%
5	0,0%	0,0%	0,0%	0,0%	0,0%	0,1%	0,2%	0,5%	0,9%	1,5%	2,5%	3,6%	5,1%	6,9%	9,0%	11,3%	14,0%	16,8%	19,9%	23,1%	26,5%	30,0%	33,5%	37,1%	40,6%
6	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,1%	0,2%	0,5%	0,8%	1,3%	1,9%	2,7%	3,8%	5,0%	6,5%	8,2%	10,2%	12,3%	14,7%	17,2%	20,0%	22,8%
7	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,1%	0,1%	0,2%	0,4%	0,7%	1,0%	1,5%	2,1%	2,8%	3,7%	4,8%	6,1%	7,5%	9,1%	10,9%
8	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,1%	0,1%	0,2%	0,3%	0,5%	0,8%	1,1%	1,6%	2,1%	2,8%	3,5%	4,5%
9	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,1%	0,1%	0,2%	0,3%	0,4%	0,6%	0,9%	1,2%	1,6%
10	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,1%	0,1%	0,2%	0,2%	0,3%	0,5%
11	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,1%	0,1%	0,1%
12	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%
13	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%
14	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%
15	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%
16	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%
17	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%
18	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%
19	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%
20	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%
21	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%
22	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%
23	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%
24	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%
25	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%

Figure 1: proba of having k identical pako only over r total dies

represent the proba of having k identical dices out of r total dices supposing we choose the best spot and pako can represent any number

k/r	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
1	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%
2	0%	44%	72%	91%	98%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%
3	0%	0%	17%	40%	64%	81%	91%	96%	99%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%
4	0%	0%	0%	6%	19%	37%	56%	71%	83%	91%	96%	98%	99%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%
5	0%	0%	0%	0%	2%	8%	19%	33%	48%	63%	75%	85%	91%	95%	98%	99%	100%	100%	100%	100%	100%	100%	100%	100%	100%
6	0%	0%	0%	0%	0%	1%	3%	9%	18%	29%	42%	56%	68%	78%	85%	91%	95%	97%	99%	99%	100%	100%	100%	100%	100%
7	0%	0%	0%	0%	0%	0%	0%	1%	4%	9%	16%	26%	37%	49%	61%	71%	79%	86%	91%	95%	97%	98%	99%	100%	100%
8	0%	0%	0%	0%	0%	0%	0%	0%	0%	2%	4%	8%	15%	23%	33%	43%	54%	64%	73%	81%	87%	91%	94%	97%	98%
9	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	1%	2%	4%	8%	13%	20%	29%	38%	48%	58%	67%	75%	82%	87%	91%
10	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	1%	2%	4%	7%	12%	18%	25%	34%	43%	52%	61%	69%	76%
11	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	1%	2%	4%	7%	11%	16%	22%	30%	38%	47%	55%
12	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	1%	2%	3%	6%	9%	14%	20%	26%	34%
13	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	1%	2%	3%	5%	8%	12%	17%
14	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	1%	2%	3%	5%	7%
15	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	1%	2%	3%
16	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	1%
17	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%
18	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%
19	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%
20	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%
21	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%
22	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%
23	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%
24	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%
25	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%

Figure 2: proba of having k identical dies over r total dies (pako included)