AOS4 : Exercise involving weighted average, value functions and Python

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January 7, 2023

After graduating from UTC, a friend of yours receive countless work offers that she wants to choose from. She does not have the time to rank them all one by one, but she quickly evaluated them with 3 criteria : salary, interest in the work, and social value. Both the interest and social value are rated on a linear scale of 1 (worst) to 5 (best). Some of the offers are written in table 1. You have to build a model to help her find the job she would have preferred had she had the time to review them all extensively.

	Salary (1000 €)	Interest	Perceived social value
O_1 (Non-profit organization)	1.2	3	4
O_2 (Developer in a small company)	2.5	2	2
O_3 (Consulting)	5	2	1
O_4 (Career in finance)	7	3	1

Table 1: Four examples of work choices

1. You first decide to use a weighted average model G, with weights $w_i \geq 0$, $i \in \{1, 2, 3\}$, and $\sum_i w_i = 1$.

   (a) Write a class `WeightedAvgModel` that has a constructor with an argument `weights`. Add a `predict` method that returns the correct index ordering of the choices given a $n * 3$ numpy array `offers` with one row being one work offer.

   (b) You ask your friend which weights she would assign to each criteria. After briefly thinking about it, she gives you the following values : $w_1 = 0.5$, $w_2 = 0.25$, $w_3 = 0.25$, as she finds the salary to be the most important criteria, and the other two to be equally important. Use these values and the code you wrote to compute the ordering of these offers.

   (c) Make sure your model never places an offer O Pareto-dominated by another offer O' higher than O'.

   (d) Imagine you have a new offer in an oil company : $O_5 = (12, 1, 1)$. Find its position in the ordering according to the model we have.

   (e) Your friend is thankful for the time you spent on this model, but she is not at all satisfied by the best choice your model finds. She actually prefers every choice to the one your model picked. She allows you to modify the salary weight w_1 and redistribute the value taken evenly between the other weights, so as to end up with another best choice. What is the lowest value (up to a precision of 10^{-4}) w_1 can have such that the best choice is no longer the one you previously had ? Compute the ordering found by this new model with your code.

   (f) A new offer O_6 pops up : it is a very interesting (unpaid) work in a small political party working on social issues your friend is passionate about, that has the following values : 0, 5, 5. Compute the ordering with this offer taken into account with the set of weights you computed in the previous question and verify it is ranked as the best option.

(g) Your friend sees what your model outputs and is again skeptical. She says she definitely does not want to do this work over the others because it would then be impossible for her to live, since as a lot of people she needs *some* money to pay for food and rent. Show your model is not able to satisfy her constraints, namely that she views interest and perceived social value as equally interesting, and that she prefers O_i to O_5 and O_6, for $i \in \{1, 2, 3, 4\}$.

You decide to switch for a slightly more complex and expressive model, namely an MAVT model :

$$V(x) = \sum_i w_i v_i(x_i) \text{ with } v_i \text{ taking values in [0,1]}, w_i \geq 0, \sum_i w_i = 1$$

The functions v_i are non-decreasing and twice differentiable.

2. (a) Your friend told you she needs some money to live but does not need as much money as is offered in O_5. What economic law does this remind you of ? What general property should the function v_1 have to respect her preference salary-wise ?

(b) Since the salary can only be positive (or 0), you decide that : $v_1(x) = 10(\frac{1}{1+e^{-ax}} - 0.5)$, which is a linear transformation (for comparison with the other criteria) of the sigmoid function with an extra parameter $a > 0$. What is the range of output of v_1 on $\mathbf{R}^+$? Verify that this function satisfies the property formulated in the previous question on $\mathbf{R}^+$.

(c) Implement this function with arguments x and a in Python so that it works on a numpy array of dimension one, without any loop. Plot the function on [0,10] with several values of a. What does a control ?

(d) Your friend tells you she is not that interested in salary increases after 4000€. Empirically find a value of a that would suit this preference and compute with the initial set of weights the ordering with this new information (keep the identity function for v_2 and v_3 : $v_2(x) = v_3(x) = x$).