Exercise Text

1 First part

Consider a lottery with three possible outcomes: $100 will be received with a probability of 0.1, $50 with a probability of 0.2, and $10 with a probability of 0.7.

1 What is the expected value of the lottery?

2 Suppose to have a ticket price lower than the expected value, would you play this lottery?

2 Second part

A casino offers a game of chance for a single player in which a fair coin is tossed at each stage. The initial stake begins at 2 dollars and is doubled every time heads appear. The first time tails appear, the game ends and the player wins whatever is the current stake. Thus the player wins 2 dollars if tails appear on the first toss, 4 dollars if heads appear on the first toss and tails on the second, 8 dollars if heads appear on the first two tosses and tails on the third, and so on.

1 How much does the player win after k consecutive head tosses? Answer with a generic formula

2 Calculate the expected value

3 Assume that the ticket cost is $40, would play this game? How much would you pay at maximum? Write an explanation about your answer

This exercise reveals the disconnect between human intuition for expected value and the mathematical notion of expected value.

Suppose there are two casinos. The first has a very little vault and can pay at a maximum of $64, and the second has a very big vault and can pay at a maximum of $1.075.000.000. This means that if you reach a consecutive number of head tosses the casino must stop the game and pay you the max available premium.

4 Which is the maximum number of coin tosses for these two values **W**[64, 1.075.000.000]? Derive a generic formula.

Now the expected value is no more infinite but depends on **W**

5 Calculate the expected value knowing L(64). Where L(x) is the maximum number of tosses given a max available premium of x

6 Calculate the expected value knowing L(1075000000)

7 Would you pay a ticket now? Is it more reasonable than before?

7 How rapidly does the maximum premium increase with respect to the resources of the casino **W**? And the average premium? Draw conclusions from the expected values just calculated